

Write yours and your teacher's name at the top of your answer sheets.

U6 Mathematics Mock

Paper 2 (Y – KRA/RLB/WBH)

February 2023

2022-2023

Duration: 1 hour 30 minutes

Total number of marks: 71

Write your answers on file paper.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

1.

Solve the equation $|2x - 3| = 9$.

[3]

2.

An artist is creating a design for a large painting. The design includes a set of steps of varying heights. In the painting the lowest step has height 20 cm and the height of each other step is 5% less than the height of the step immediately below it.

In the painting the total height of the steps is 205 cm, correct to the nearest centimetre.

Determine the number of steps in the design.

[5]

3.

(a) Give full details of the single transformation that transforms the graph of $y = x^3$ to the graph of $y = x^3 - 8$. [2]

The function f is defined by $f(x) = x^3 - 8$.

(b) Find an expression for $f^{-1}(x)$. [2]

(c) State how the graphs of $y = f(x)$ and $y = f^{-1}(x)$ are related geometrically. [1]

4.

The positive integers x , y and z are the first, second and third terms, respectively, of an arithmetic progression with common difference -4 .

Also, x , $\frac{15}{y}$ and z are the first, second and third terms, respectively, of a geometric progression.

(a) Show that y satisfies the equation $y^4 - 16y^2 - 225 = 0$. [4]

(b) Hence determine the sum to infinity of the geometric progression. [4]

5.

(i) Given that p and q are integers such that

$$pq \text{ is even}$$

use algebra to prove by contradiction that at least one of p or q is even.

(3)

(ii) Given that x and y are integers such that

- $x < 0$
- $(x + y)^2 < 9x^2 + y^2$

show that $y > 4x$

(2)

6.

In this question you must show detailed reasoning.

(a) Show that the equation $m \sec \theta + 3 \cos \theta = 4 \sin \theta$ can be expressed in the form

$$m \tan^2 \theta - 4 \tan \theta + (m + 3) = 0. \quad [3]$$

(b) It is given that there is only one value of θ , for $0 < \theta < \pi$, satisfying the equation $m \sec \theta + 3 \cos \theta = 4 \sin \theta$.

Given also that m is a negative integer, find this value of θ , correct to 3 significant figures.

[5]

7.

(a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$\sqrt{4 - 9x}$$

writing each term in simplest form.

(4)

A student uses this expansion with $x = \frac{1}{9}$ to find an approximation for $\sqrt{3}$

Using the answer to part (a) and without doing any calculations,

(b) state whether this approximation will be an overestimate or an underestimate of $\sqrt{3}$ giving a brief reason for your answer.

(1)

8.

(a) Find the first two terms, in ascending powers of x , of the binomial expansion of

$$\left(1 - \frac{x}{2}\right)^{\frac{1}{2}}$$

[2 marks]

(b) Hence, for small values of x , show that

$$\sin 4x + \sqrt{\cos x} \approx A + Bx + Cx^2$$

where A , B and C are constants to be found.

[4 marks]

9.

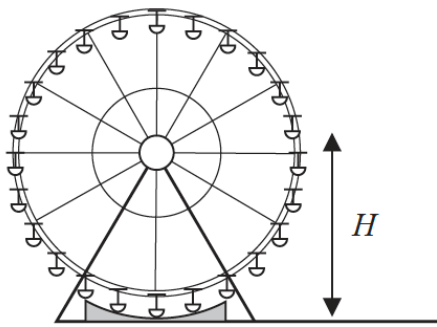


Figure 4

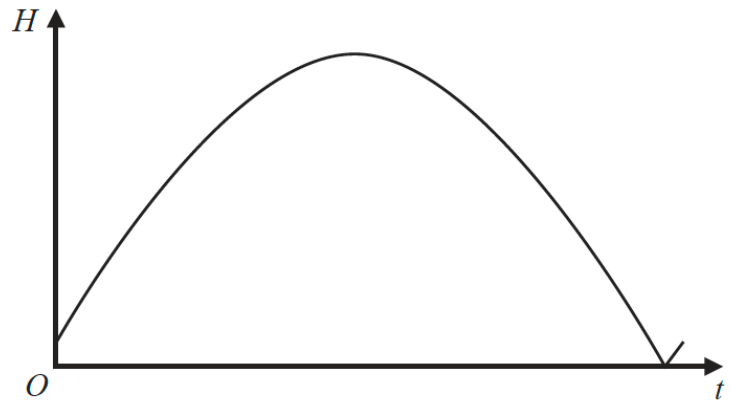


Figure 5

Figure 4 shows a sketch of a Ferris wheel.

The height above the ground, H m, of a passenger on the Ferris wheel, t seconds after the wheel starts turning, is modelled by the equation

$$H = |A \sin(bt + \alpha)^\circ|$$

where A , b and α are constants.

Figure 5 shows a sketch of the graph of H against t , for one revolution of the wheel.

Given that

- the maximum height of the passenger above the ground is 50 m
- the passenger is 1 m above the ground when the wheel starts turning
- the wheel takes 720 seconds to complete one revolution

(a) find a complete equation for the model, giving the exact value of A , the exact value of b and the value of α to 3 significant figures.

(4)

(b) Explain why an equation of the form

$$H = |A \sin(bt + \alpha)^\circ| + d$$

where d is a positive constant, would be a more appropriate model.

(1)

10.

A particle P moves along a straight line in such a way that at time t seconds P has velocity $v \text{ m s}^{-1}$, where

$$v = 12 \cos t + 5 \sin t.$$

- (a) Express v in the form $R \cos(t - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the value of α correct to 4 significant figures. [3]
- (b) Hence find the two smallest positive values of t for which P is moving, in either direction, with a speed of 3 m s^{-1} . [3]

11.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that

$$2 \sin(x - 60^\circ) = \cos(x - 30^\circ)$$

show that

$$\tan x = 3\sqrt{3} \quad (4)$$

(b) Hence or otherwise solve, for $0 \leq \theta < 180^\circ$

$$2 \sin 2\theta = \cos(2\theta + 30^\circ)$$

giving your answers to one decimal place.

(4)

Statistics

12.

There are 25 students in a class.

- The number of students who study both History and English is 3.
- The number of students who study neither History nor English is 14.
- The number of students who study History but not English is three times the number who study English but not History.

- (a) • Show this information on a Venn diagram.
• Determine the probability that a student selected at random studies English. [4]

Two different students from the class are chosen at random.

- (b) Given that exactly one of the two students studies English, determine the probability that exactly one of the two students studies History. [6]