Write yours and your teacher's name at the top of your answer sheets.

U6 Mathematics Mock

Paper 2 (Teacher Y)

February 2024

2023-2024

Duration: 1 hour 30 minutes

Total number of marks: 75

Write your answers on file paper.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \dfrac{u}{v}$, $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)\big(f(x)\big)^n\,dx = \frac{1}{n+1}\big(f(x)\big)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \dbinom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

1.

The first, third and fourth terms of an arithmetic progression are u_1, u_3 and u_4 respectively, where

$$u_1 = 2\sin\theta, \qquad u_3 = -\sqrt{3}\cos\theta, \qquad u_4 = \tfrac{7}{2}\sin\theta,$$

and $\frac{1}{2}\pi < \theta < \pi$.

(a) Determine the exact value of θ. **[3]**

(b) Hence determine the value of $\sum_{r=1}^{100} u_r$. **[3]**

2.

(a) Express $3\sin x - 4\cos x$ in the form $R\sin(x-\alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of α correct to **4** significant figures. **[3]**

(b) Hence solve the equation $3\sin x - 4\cos x = 2$ for $0° < x < 90°$, giving your answer correct to **3** significant figures. **[2]**

3.

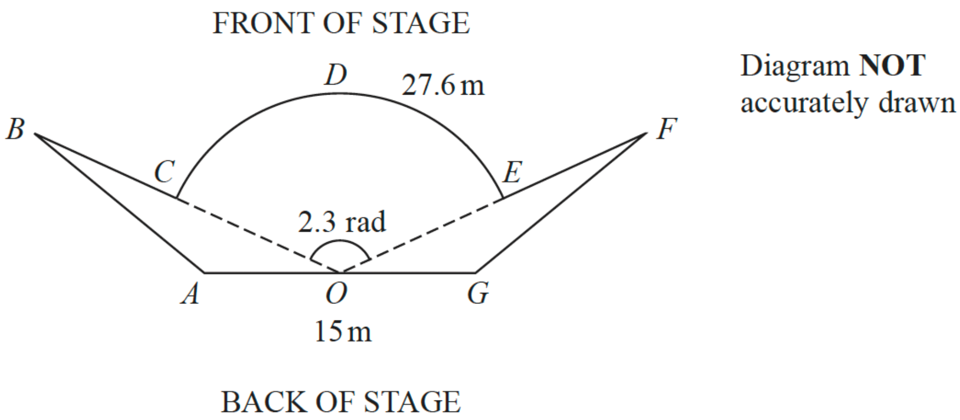

Figure 1

Figure 1 shows the plan view of a stage.

The plan view shows two congruent triangles *ABO* and *GFO* joined to a sector *OCDEO* of a circle, centre *O*, where

- angle COE = 2.3 radians
- arc length CDE = 27.6 m
- AOG is a straight line of length 15 m

(a) Show that OC = 12 m. **(2)**

(b) Show that the size of angle AOB is 0.421 radians correct to 3 decimal places. **(2)**

Given that the total length of the front of the stage, *BCDEF*, is 35 m,

(c) find the total area of the stage, giving your answer to the nearest square metre. **(6)**

4.

A student wishes to prove that, for all positive integers a and b, $a^2 - 4b \neq 2$.

(a) Prove that $a^2 - 4b = 2 \Rightarrow a$ is even. **[2]**

(b) Hence or otherwise prove that, for all positive integers a and b, $a^2 - 4b \neq 2$. **[3]**

5.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\tan\theta(8\cos\theta + 23\sin^2\theta) = 8\sin 2\theta(1 + \tan^2\theta)$$

may be written as

$$\sin 2\theta(A\cos^2\theta + B\cos\theta + C) = 0$$

where A, B and C are constants to be found.

(3)

(b) Hence, solve for $360° \leqslant x \leqslant 540°$

$$2\tan x(8\cos x + 23\sin^2 x) = 8\sin 2x(1 + \tan^2 x) \qquad x \in \mathbb{R} \qquad x \neq 450°$$

(4)

6.

(a) The function $f(x)$ is defined for all values of x as $f(x) = |ax - b|$, where a and b are positive constants.

(i) The graph of $y = f(x) + c$, where c is a constant, has a vertex at (3, 1) and crosses the y-axis at (0, 7).

Find the values of a, b and c. **[3]**

(ii) Explain why $f^{-1}(x)$ does not exist. **[1]**

(b) The function $g(x)$ is defined for $x \geqslant \frac{q}{p}$ as $g(x) = |px - q|$, where p and q are positive constants.

(i) Find, in terms of p and q, an expression for $g^{-1}(x)$, stating the domain of $g^{-1}(x)$. **[3]**

(ii) State the set of values of p for which the equation $g(x) = g^{-1}(x)$ has no solutions. **[1]**

7.

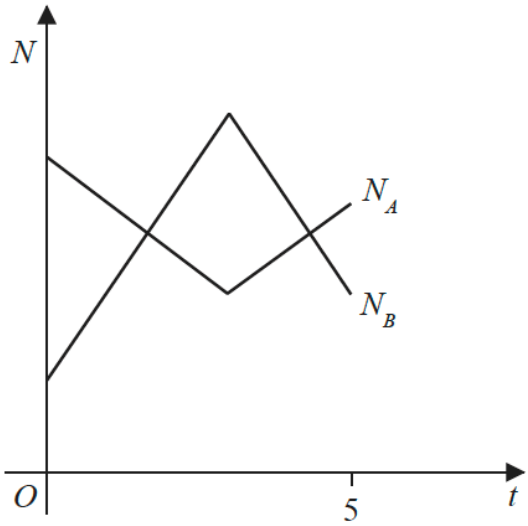

Figure 2

The number of subscribers to two different music streaming companies is being monitored.

The number of subscribers, N_A, in thousands, to **company A** is modelled by the equation

$$N_A = |t - 3| + 4 \qquad t \geqslant 0$$

where t is the time in years since monitoring began.

The number of subscribers, N_B, in thousands, to **company B** is modelled by the equation

$$N_B = 8 - |2t - 6| \qquad t \geqslant 0$$

where t is the time in years since monitoring began.

Figure 2 shows a sketch of the graph of N_A and the graph of N_B over a 5-year period.

Use the equations of the models to answer parts (a), (b), (c) and (d).

(a) Find the initial difference between the number of subscribers to **company A** and the number of subscribers to **company B**.

(2)

When $t = T$ **company A** reduced its subscription prices and the number of subscribers increased.

(b) Suggest a value for T, giving a reason for your answer.

(2)

(c) Find the range of values of t for which $N_A > N_B$ giving your answer in set notation.

(5)

8.

(a) Find the first **three** terms in the expansion of $(4+3x)^{\frac{3}{2}}$ in ascending powers of x. **[4]**

(b) State the range of values of x for which the expansion in part **(a)** is valid. **[1]**

(c) In the expansion of $(4+3x)^{\frac{3}{2}}(1+ax)^2$ the coefficient of x^2 is $\frac{107}{16}$.

Determine the possible values of the constant a. **[4]**

9.

(a) Use the result $\cos(A+B) = \cos A\cos B - \sin A\sin B$ to show that

$\cos(A-B) = \cos A\cos B + \sin A\sin B$. **[2]**

The function $f(\theta)$ is defined as $\cos(\theta+30°)\cos(\theta-30°)$, where θ is in degrees.

(b) Show that $f(\theta) = \cos^2\theta - \frac{1}{4}$. **[3]**

(c) **(i)** Determine the following.

- The **maximum** value of $f(\theta)$
- The smallest **positive** value of θ for which this maximum value occurs **[2]**

(ii) Determine the following.

- The **minimum** value of $f(\theta)$
- The smallest **positive** value of θ for which this minimum value occurs **[2]**

Statistics

10.

In this question you must show detailed reasoning.

A disease that affects trees shows no visible evidence for the first few years after the tree is infected.

A test has been developed to determine whether a particular tree has the disease. A positive result to the test suggests that the tree has the disease. However, the test is not 100% reliable, and a researcher uses the following model.

- If the tree has the disease, the probability of a positive result is 0.95.
- If the tree does not have the disease, the probability of a positive result is 0.1.

(a) It is known that in a certain county, A, 35% of the trees have the disease. A tree in county A is chosen at random and is tested.

Given that the result is positive, determine the probability that this tree has the disease. **[3]**

A forestry company wants to determine what proportion of trees in another county, B, have the disease. They choose a large random sample of trees in county B.

Each tree in the sample is tested and it is found that the result is positive for 43% of these trees.

(b) By carrying out a calculation, determine an estimate of the proportion of trees in county B that have the disease. **[4]**