U6 Mock Teacher Y 23-24 SOLUTIONS [75]

1.

(a)	$u_1 = a = 2\sin\theta, u_3 = a + 2d = -\sqrt{3}\cos\theta$ and $u_4 = a + 3d = \frac{7}{2}\sin\theta$			**For reference**	
	$d = \frac{7}{2}\sin\theta + \sqrt{3}\cos\theta$	**B1***	**2.1**	Forming a correct expression for d (or a correct equation containing d) e.g. $(d =)\frac{1}{2}(-\sqrt{3}\cos\theta - 2\sin\theta)$, $(d =)\frac{1}{3}(\frac{7}{2}\sin\theta - 2\sin\theta)(= 0.5\sin\theta)$ Can be implied e.g. $\frac{7}{2}\sin\theta = 2\sin\theta + 3(...)$ seen	Can be implied by a correct equation for θ
	$-\sqrt{3}\cos\theta = 2\sin\theta + 2(\frac{7}{2}\sin\theta + \sqrt{3}\cos\theta) \Rightarrow$ $\tan\theta = -\frac{\sqrt{3}}{3}$	**M1dep***	**3.1a**	Obtaining an equation of the form $\tan\theta = k$ from a trigonometric equation which initially had 3 sine and 1 cosine terms **or** 2 sine and 2 cosine terms e.g. **if** correct $\frac{7}{2}\sin\theta = 2\sin\theta + 3(\frac{7}{2}\sin\theta + \sqrt{3}\cos\theta)$	
	$\theta = \frac{5}{6}\pi$	**A1**	**2.2a**	Condone $-\frac{\pi}{6}$ stated too but **A0** if any other value given in the interval $\frac{1}{2}\pi < \theta < \pi$ (but ignore any values that are given outside this range)	Exact answer must be seen at some stage
		[3]			
(b)	$S_{100} = \frac{100}{2}[2(2\sin\theta) + (100-1)d]$	**B1ft**	**1.2**	Correct formula for the sum of an AP with $a = 2\sin\theta$ (with either θ or their value of θ substituted) **and** either d **or** their value of d substituted **or** their expression for d	Follow through their values of θ and d if used provided $\frac{100}{2}[2(2\sin\theta) + (100-1)d]$ implied
	$d = \frac{7}{2}\sin(\frac{5}{6}\pi) + \sqrt{3}\cos(\frac{5}{6}\pi)$ $(= \frac{1}{4})$	**B1ft**	**1.1**	Correct expression for d **using their** θ (e.g. $d = \frac{1}{2}(-\sqrt{3}\cos\theta - 2\sin\theta)$, $d = \frac{1}{3}(\frac{7}{2}\sin\theta - 2\sin\theta)$)	Follow through their value of θ **only**
	$S_{100} = 1337.5$	**B1**	**2.2a**	**www** – must have come from $\theta = \frac{5}{6}\pi$ correctly derived in **(a)** oe (not for 1338 or 1340 unless 1337.5 seen so isw once 1337.5 (oe e.g. $\frac{2675}{2}$) seen)	**Correct answer with no working scores all 3 marks**
		[3]			

2.

(a)	$R=5$	B1	1.1	**B0** for $R=\pm5,\sqrt{25}$ etc. unless replaced with 5	No working required for this mark. **Ignore working**
	$R\cos\alpha=3$, $R\sin\alpha=4 \Rightarrow \tan\alpha=\frac{4}{3}$	M1	1.1	**M1** for $\tan\alpha=k$ where $k=\pm\frac{3}{4},\pm\frac{4}{3}$ **or** equivalent e.g. $\cos\alpha=\pm\frac{3}{R},\ \sin\alpha=\pm\frac{4}{R}$ **with** their value of R (but not just R and do not allow reciprocals for this mark). 53.1 (or better) with no working implies **M1**	**SC** If $\cos\alpha=3, \sin\alpha=4$ $\Rightarrow \tan\alpha=\frac{4}{3}$ explicitly seen then this scores **M1 A0** but do not penalise again in **(b)** (if correct answer seen)
	$\alpha=53.13$	A1	1.1	**www** awrt 53.13 (at least **4** sf required) so 53.1 (or 53) is **A0** (but if an awrt 53.13 seen then isw if replaced with a less accurate value)	53.13010235… - an answer in radians scores **A0** 53.13 from $R\sin(x-\alpha)$ soi
		[3]			
(b)	$x=53.13+\arcsin\left(\frac{2}{5}\right)$	M1	1.1	M1 for $x=\alpha+\arcsin\left(\frac{2}{R}\right)$ or $x-\alpha=\arcsin\left(\frac{2}{R}\right)$ with their R and α substituted	**SC B1** for 76.7 only (in the given range) from using an alternative method e.g. $9\sin^2 x=(2+4\cos x)^2$
	$x=76.7$	A1	1.1	awrt 76.7 (at least **3** sf required) – ignore any answers given outside the range $0<x<90$ but do not award this mark if any other values in this range are given – **www but see SC in (a)**	**Correct answer with no working seen scores SC B1** Answer in radians scores **A0**
		[2]			

3.

3(a)	$OC\times2.3=27.6$	M1	1.1b
	e.g. $OC=\frac{27.6}{2.3}=12\text{ m}$ *	A1*	2.1
		(2)	
(b)	e.g. $(2AOB=)\ \pi-2.3$	M1	1.1b
	$\frac{\pi-2.3}{2}\Rightarrow 0.421\ \text{rad}$ *	A1*	2.1
		(2)	
(c)	Area $OCDE=\frac{1}{2}\times12^2\times2.3$	M1	1.1b
	$=165.6\ (\text{m}^2)$ (accept awrt 166)	A1	1.1b
	$(OB=)\ \frac{35-27.6}{2}+12=15.7\text{ m}$	B1	2.1
	Area of OAB (or OFG) $=\frac{1}{2}\times"15.7"\times7.5\times\sin0.421\quad(=24.0...\text{m}^2)$	M1	1.1b
	Total area = "165.6"+2×"24.1"	dM1	3.1a
	= awrt 214 (m²)	A1	1.1b
		(6)	

4.

(a)		$a^2 = 4b + 2$	M1	2.1	Setting up so that the deduction a^2 is even can be made.
		Hence a^2 is even. Hence a is even	A1	2.2a	www, must see both statements and a convincing, correct, argument oe (e.g. $a^2 = 2(2b + 1)$)
		Alternative method Assume that a is odd, then a^2 is odd	M1		For setting up and stating that a is odd $\Rightarrow a^2$ is odd May see (not required) $a = 2n+1$, $a^2 = 2(2n^2+2n)+1$ Hence a^2 is odd
		$4b$ is even, so $a^2 - 4b$ is odd Hence 2 is odd (so contradiction) Hence a is even.	A1		www, Must see both statements and a convincing, correct, argument
			[2]		
(b)		Assume that $a^2 - 4b = 2$ Let $a = 2n$, (where n is an integer)	M1	2.1	Setting up (must see assumption and use of a is even)
		Either of: $4n^2 - 4b = 2$ / $2n^2 - 2b = 1$, | $4n^2 - 4b = 2$ / $n^2 - b = 0.5$,	A1	2.1	Substituting in $a = 2n$ and correctly reaching an equation which shows a contradiction. Accept the equivalent in words if clear and correct. Also accept: $4n^2 - 4b$ is a multiple of 4, Hence $a^2 - 4b$ is a multiple of 4, which is a contradiction
		Hence 1 is even | $n^2 - b$ is an integer (Contradiction) Hence $a^2 - 4b \neq 2$	A1	2.2a	www, Must see both statements and a convincing, correct, argument
		Alternative method (1) Assume that $a^2 - 4b = 2$ $\Rightarrow a^2 = 4b + 2 = 2(2b + 1)$	M1		
		For a to be an integer, $2b + 1$ must be even But ($2b$ is even, so) $2b + 1$ is odd	A1		
		(Which is a contradiction) hence $a^2 - 4b \neq 2$	A1		Must see both statements and a convincing, correct, argument www
		Alternative method (2) a even $\Rightarrow a^2 = 4n$ (n an integer) $\Rightarrow a^2$ is congruent to 0 mod 4	M1		
		$4b + 2$ is congruent to 2 mod 4	A1		
		Therefore a^2 cannot equal $4b + 2$	A1		Must see previous two lines and a convincing, correct, argument www
		Alternative method (3) Assume that $a^2 - 4b = 2$, then a is even (and consider whether b is odd or even)	M1		For setting up using part **(a)** and considering either case where b is odd or even (ignore any reference to the cases where a is odd as these are not required) May see (but not required) $a=2n$ so $a^2=4n^2$ Condone using the same letter (e.g. n) in a and b for this mark only.
		If b is odd then 2 is either 0 or a multiple of 4, (so contradiction) AND If b is even then 2 is either 0 or a multiple of 4, (so contradiction)	A1		For correctly considering both cases either algebraically or in words. May see (but not required) $b = 2m+1$, so $a^2 - 4b = 4(n^2 - (2m+1))$ And $b = 2m$, so $a^2 - 4b = 4(n^2 - 2m)$ Do not award this mark if same integer (e.g. n) used in both a and b
		Therefore a^2 cannot equal $4b + 2$	A1		A fully correct, convincing argument with conclusion, www.
			[3]		

5.

4(a)	e.g. $2\frac{\sin\theta}{\cos\theta}(8\cos\theta + 23\underline{(1-\cos^2\theta)}) = 8\times\underline{2\sin\theta\cos\theta\sec^2\theta}$	B1
	$2\tan\theta(8\cos\theta + 23\sin^2\theta) = 8\sin 2\theta\sec^2\theta$ $\Rightarrow 2\sin\theta\cos\theta(8\cos\theta + 23(1-\cos^2\theta)) = 8\sin 2\theta$ $\sin 2\theta(8\cos\theta + 23(1-\cos^2\theta)) = 8\sin 2\theta$ $\sin 2\theta(23\cos^2\theta - 8\cos\theta - 15) = 0$	M1A1
		(3)
(b)	$\sin 2x(23\cos^2 x - 8\cos x - 15) = 0$	
	$\sin 2x = 0 \Rightarrow x = 360°$ **or** $540°$	B1
	$23\cos^2 x - 8\cos x - 15 \Rightarrow \cos x = -\frac{15}{23}$	M1
	$\cos x = -\frac{15}{23} \Rightarrow x = \ldots$	dM1
	$x = 360°$, $540°$ and awrt $491°$ only	A1
		(4)

6.

(a)	**(i)**	$a = 2$	**B1**	**1.1**	Either stated or embedded in equation	eg $\|2x - b\|$ seen ignore any other values seen **B0** for $a = -2$, unless subsequently corrected
		$b = 6$	**B1**	**1.1**	Either stated or embedded in equation	eg $\|ax - 6\|$ seen ignore any other values seen
		$c = 1$	**B1**	**1.1**	Either stated or embedded in equation	eg $\|ax - b\| + 1$ seen ignore any other values seen
			[3]			
(a)	**(ii)**	Because f is a many to one function eg f(0) = f(6)	**B1**	**1.2**	Any correct reason	Condone no explicit example Could also say 'because f is not one to one' **B1** BOD for 'it is not one to one' If referring to 'one to many' or 'many to one' it must be clear whether this is f or f^{-1} (just 'it' or 'the function' is not enough) Allow implication of function eg 'as it is a many to one function there is no inverse function' May also refer to the 'horizontal line test', but need to state outcome eg horizontal line would cross graph of $y = f(x)$ twice
			[1]			
(b)	**(i)**	$y = px - q$ $px = y + q$ $x = \frac{1}{p}(y + q)$	**M1**	**3.1a**	Complete attempt to find inverse function of $f(x) = px - q$	Correct order of operations, allow sign error only Could use coordinate geometry and reflection in $y = x$ Allow **M1** BOD if more than one function is being considered
		$g^{-1}(x) = \frac{1}{p}x + \frac{q}{p}$	**A1**	**1.1**	Obtain correct inverse, in terms of x	Could be single term ie $g^{-1}(x) = \frac{x+q}{p}$ **A1** for just $\frac{1}{p}x + \frac{q}{p}$, ie $g^{-1}(x)$ can be omitted If LHS seen, it must be $g^{-1}(x)$ or y (allow BOD for g^{-1}, or using f not g) BOD if modulus sign included **A0** if additional equations given
		$x \geq 0$	**B1**	**1.2**	Correct domain **B0** for $x > 0$	Independent of the first two marks If in words then must be correct, so **B1** for 'any non-negative x' but **B0** for 'any positive x' $g^{-1}(x) \geq 0$ is **B0** Condone incorrect set notation as long as intention is clear
			[3]			
(b)	**(ii)**	$0 < p \leq 1$	**B1**	**3.1a**	Correct set of values, any notation No need for $0 < p$ as specified in question, so **B1** for $p \leq 1$	**B0** for $p < 1$ **B0** for any additional incorrect values **B0** if just single example and not set of values Condone incorrect set notation as long as intention is clear
			[1]			

7.

2(a)	$N_A - N_B = (3+4)-(8-6) = \ldots$	**M1**
	5000 (subscribers)	**A1**
		(2)
(b)	$(T=)3$	**B1**
	This was the point when company A had the lowest number of subscribers	**B1**
		(2)
(c)		
	$-t+7=2t+2$ o.e. or $t+1=14-2t$ o.e.	**B1**
	$-t+7=2t+2$ o.e. $\Rightarrow t=\ldots$ or $t+1=14-2t$ o.e. $\Rightarrow t=\ldots$	**M1**
	One of the two critical values $t=\frac{5}{3}$ or $t=\frac{13}{3}$	**A1**
	Chooses the outside region for their **two** values of t Both of $t<"\frac{5}{3}"$, $t>"\frac{13}{3}"$	**A1ft**
	$\left\{t\in\mathbb{R}: t<\frac{5}{3}\right\}\cup\left\{t\in\mathbb{R}: t>\frac{13}{3}\right\}$	**A1**
		(5)

8.

(a)		$(1+\frac{3}{4}x)^{\frac{3}{2}} = 1+(\frac{3}{2})(\frac{3}{4}x)$	**B1**	**1.1**	Correct first two terms	Allow unsimplified Expect $1+\frac{9}{8}x$
		$+\frac{(\frac{3}{2})(\frac{1}{2})}{2}(\frac{3}{4}x)^2$	**M1**	**1.1**	Attempt third term	Condone lack of brackets when attempting to square ie $\frac{3}{4}x^2$ Coefficient must be $\frac{(\frac{3}{2})(\frac{1}{2})}{2}$ or equiv
			A1	**1.1**	Obtain correct third term	Allow unsimplified $\frac{3}{4}x^2$ is **A0** unless recovered by later work Expect $\frac{27}{128}x^2$
		$(4+3x)^{\frac{3}{2}} = 8(1+\frac{3}{4}x)^{\frac{3}{2}} = 8+9x+\frac{27}{16}x^2$	**B1FT** **[4]**	**1.1a**	Multiply their 3 term expansion by 8	Bracket expanded and coefficients simplified If **B1M1A1** awarded, but attempt to simplify then goes wrong, **B1FT** is **not** also awarded ISW once correct expansion seen
(b)		$\lvert x\rvert<\frac{4}{3}$ or $-\frac{4}{3}<x<\frac{4}{3}$	**B1** **[1]**	**1.1**	Could also be $\lvert x\rvert\le\frac{4}{3}$ or $-\frac{4}{3}\le x\le\frac{4}{3}$, as $n>0$	Must be condition for x, not kx
(c)		$(8+9x+\frac{27}{16}x^2)(1+2ax+a^2x^2)$ coeff of x^2 is $8a^2+18a+\frac{27}{16}$	**M1**	**3.1a**	Expand $(1+ax)^2$ and attempt at least one coeff of x^2	Allow ax as middle term, and/or ax^2 as third term Attempt at x^2 term could be part of a fuller expansion
			M1	**1.1**	Attempt all three coeff of x^2, and no others	If part of fuller expansion then **M1** awarded when only three relevant terms used
		$8a^2+18a+\frac{27}{16}=\frac{107}{16}$ $8a^2+18a-5=0$	**A1**	**3.1a**	Equate to $\frac{107}{16}$ to obtain correct quadratic	aef, including unsimplified **A0** if a mix of terms and coefficients, but can be recovered
		$(2a+5)(4a-1)=0$ $a=-\frac{5}{2}$ and $a=\frac{1}{4}$	**A1** **[4]**	**1.1**	Solve quadratic, possibly **BC**, to obtain $a=-\frac{5}{2}$ and $a=\frac{1}{4}$	

9.

(a)		$\cos(A-B) = \cos A\cos(-B) - \sin A\sin(-B)$	M1	2.1	Replace B with $-B$ in given identity	
		$\cos(-B) = \cos B$, $\sin(-B) = -\sin B$, $\cos(A-B) = \cos A\cos B - \sin A(-\sin B)$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$ **A.G.**	A1 **[2]**	2.4	State $\cos(-B) = \cos B$ and $\sin(-B) = -\sin B$, and conclude with correct identity Condone $-\sin A\sin(-B)$ becoming $\sin A\sin B$ with no intermediate step	$\cos(-B) = \cos B$, $\sin(-B) = -\sin B$ must be stated, but no justification needed
(b)		$\left(\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta\right)\left(\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta\right)$	B1	2.1	Use correct identities, with exact trig values, to obtain a correct expression	Allow BOD for ambiguous positioning of + and – signs in a product, but penalise explicit errors if a single identity is seen in isolation If expansion done before exact trig values used, then the expression must still be correct at the point that the **B1** is awarded
		$\frac{3}{4}\cos^2\theta - \frac{1}{4}\sin^2\theta$	M1	2.1	Expand brackets May be recognised as difference of two squares so no need to see $\frac{\sqrt{3}}{4}\cos\theta\sin\theta - \frac{\sqrt{3}}{4}\cos\theta\sin\theta$	To obtain answer of form $a\cos^2\theta - b\sin^2\theta$ ($a > 0$, $b > 0$), with possibly $c\cos\theta\sin\theta - c\cos\theta\sin\theta$ also present
		$\frac{3}{4}\cos^2\theta - \frac{1}{4}(1-\cos^2\theta)$ $\cos^2\theta - \frac{1}{4}$ **A.G.**	A1 **[3]**	2.1	Use Pythagorean identity and simplify to given answer	www eg if middle terms shown for expansion, then these must be correct
(c)	(i)	max value is $\frac{3}{4}$	B1	1.1	Correct max value	
		when θ is 180°	B1 **[2]**	1.1	Correct angle	**B0** if any extra angles given Must be 'positive' so **B0** for 0° Must be in degrees Marks are independent
(c)	(ii)	min value is $-\frac{1}{4}$	B1	1.1	Correct min value	
		when θ is 90°	B1 **[2]**	1.1	Correct angle	**B0** if any extra angles given Must be in degrees **SC** If angles in **both** parts are correct, but in radians, then penalise only once (mark as **B0** in **(i)** and **B1** in **(ii)**) Marks are independent

10.

(a)		P(has disease | positive result) $= \dfrac{\text{P(has disease \& positive result)}}{\text{P(positive result)}}$	M1	3.4	Attempting this calculation, allow wrong values but for this mark must be a fraction with a product in the numerator and a sum of two products in the denominator.
		$= \dfrac{0.35\times0.95}{0.35\times0.95+0.65\times0.1}$	A1	1.1	Fully correct expression
		= 0.836 (3 sf)	A1 **[3]**	1.1	Or 133/159 or 0.8365 (4sf) (0.836477…)
(b)		(Let proportion having the disease = p) $p \times 0.95 + (1-p) \times 0.1$	M1	1.1	Setting up an expression in this form using the given values
		$p \times 0.95 + (1-p) \times 0.1 = 0.43$ $0.85p = 0.33$	M1	3.4	Setting their expression =0.43 and attempting to solve
		$p = 0.388$	A1	1.1	cao (watch for 0.389 from incorrect working)
		About 39% of trees (in county B) have the disease	B1FT **[4]**	3.2a	"Around 38.8 or 39 or 40" (oe e.g. 2/5). Must be in context and include "about" or "approximately" or "roughly" oe