Write yours and your teacher's name at the top of your answer sheets.

U6 Mathematics Mock

Paper 2 (Teacher Y)

February 2025

2024-2025

Duration: 1 hour 30 minutes

Total number of marks: 75

Write your answers on file paper.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \dfrac{u}{v}$, $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,dx = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,dx = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \tfrac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\overline{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

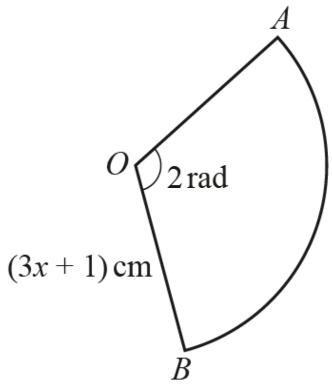
$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

1.

In this question you must show detailed reasoning.



The diagram shows a sector AOB of a circle with centre O and radius $(3x+1)$ cm. The angle AOB is 2 radians. The area of sector AOB is less than $(44x-7)\text{cm}^2$.

Find the set of possible values of x. Give your answer in set notation. **[5]**

2.

The point $P(3, -2)$ lies on the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$

Find the coordinates of the point to which P is mapped when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation

(i) $y = \mathrm{f}(x - 2)$

(ii) $y = \mathrm{f}(2x)$

(iii) $y = 3\mathrm{f}(-x) + 5$

(4)

3.

In this question you must show detailed reasoning.

The first three terms of a convergent geometric progression are $2x+3$, $x+9$ and $2x-6$ respectively.

Determine the sum to infinity of this geometric progression. **[8]**

4.

(a) Expand $(3-2x)^{-2}$ in ascending powers of x up to and including the term in x^2. **[4]**

(b) State the set of values of x for which this expansion is valid. **[1]**

(c) When $\dfrac{a+x}{(3-2x)^2}$ is expanded in ascending powers of x, the coefficient of x is zero.

Determine the value of the constant a. **[2]**

5.

Prove that there is no largest value of k in the interval $3 < k < 4$

[4 marks]

6.

The functions f and g are defined by

$$f(x) = 4 - 3x^2 \qquad x \in \mathbb{R}$$

$$g(x) = \frac{5}{2x-9} \qquad x \in \mathbb{R},\ x \neq \frac{9}{2}$$

(a) Find fg(2)

(2)

(b) Find g^{-1}

(3)

(c) (i) Find gf(x), giving your answer as a simplified fraction.

(ii) Deduce the range of gf(x).

(3)

7.

(a) Show that the equation $2\cot^2 x - 9\operatorname{cosec} x - 3 = 0$ can be expressed in the form

$5\sin^2 x + 9\sin x - 2 = 0.$ **[3]**

(b) **(i)** **In this question you must show detailed reasoning.**

Hence solve, for $0 < \theta < \pi$,

$2\cot^2 2\theta - 9\operatorname{cosec} 2\theta - 3 = 0.$

Give your answers correct to **3** decimal places. **[4]**

The small angle approximation for $\sin 2\theta$ is used to find an approximation for the smallest positive solution of the equation $2\cot^2 2\theta - 9\operatorname{cosec} 2\theta - 3 = 0$.

(ii) Show that this approximate solution is accurate to **2** decimal places. **[2]**

8.

Two arithmetic progressions, A and B, each have 100 terms denoted by a_i and b_i respectively, where $i = 1, 2, 3, \ldots 100$.

The common difference of A is d, where d is a positive integer.

The two progressions have the following properties.

- $a_1 = b_{100} = 4$
- $b_1 = a_{100}$

(a) You are given that there is at least one value of i for which $b_i = 10 + a_i$.

Show that, in this case,

$i = \frac{101}{2} - \frac{5}{d}.$ **[6]**

(b) Hence show that it is impossible for the equation $b_i = 10 + a_i$ to hold unless d takes certain values, which should be stated. **[2]**

9.

Solve the equation $\left|\frac{1}{2}x-1\right|=|2x-3|$. **[3]**

10.

In this question you must show detailed reasoning.

(a) **(i)** Use the formula for $\cos(A+B)$, and the double angle formulae, to show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **[2]**

(ii) Use this result to solve the equation $4\cos^3\theta - 3\cos\theta - \frac{\sqrt{2}}{2} = 0$ for $0° \leqslant \theta \leqslant 180°$. **[3]**

(b) **(i)** Show that $\left(x+\frac{\sqrt{2}}{2}\right)\left(4x^2-2\sqrt{2}x-1\right) = 4x^3-3x-\frac{\sqrt{2}}{2}$. **[1]**

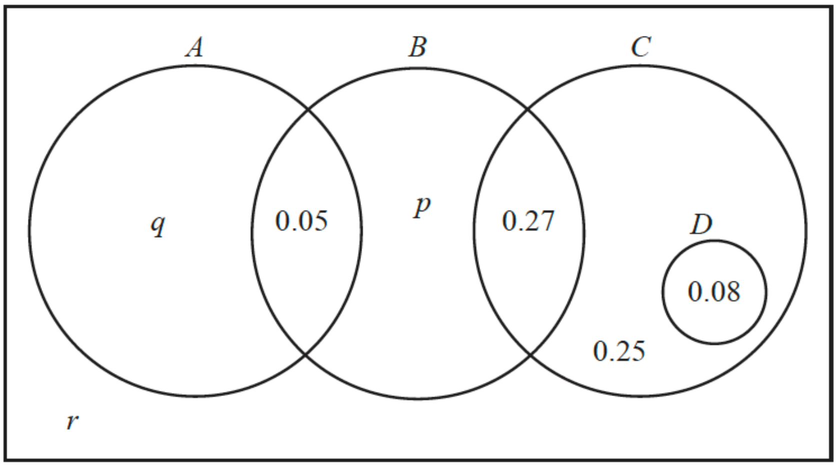
(ii) Hence find the exact roots of the equation $4x^3-3x-\frac{\sqrt{2}}{2}=0$. **[2]**

(c) Use the results from parts **(a)(ii)** and **(b)(ii)** to show that $\cos 15° = \frac{\sqrt{2}+\sqrt{6}}{4}$. **[2]**

Statistics

11.

The Venn diagram, where p, q and r are probabilities, shows the events A, B, C and D and associated probabilities.



(a) State any pair of mutually exclusive events from A, B, C and D

(1)

The events B and C are independent.

(b) Find the value of p

(2)

(c) Find the greatest possible value of $P(A \mid B')$

(3)

Given that $P(B \mid A') = 0.5$

(d) find the value of q and the value of r

(3)