xU6 Mock Teacher Y 24-25 SOLUTIONS [75]

1.

DR				**For reference only:**
$\frac{1}{2}(3x+1)^2(2)$ or $(3x+1)^2$	**B1***	**1.1**	Correct use of $A=\frac{1}{2}r^2\theta$	$\frac{1}{2}(3x+1)^2(2)<44x-7$
$9x^2+6x+1<44x-7 \Rightarrow 9x^2-38x+8\,(<0)$	**B1dep***	**1.1**	Expand and re-arrange to correct 3TQ expression in x	Allow **any** inequality sign or equals
$9x^2-38x+8(<0) \Rightarrow (9x-2)(x-4)(<0)$	**M1dep***	**1.1**	**SEE APPENDIX** for awarding this mark (solving 3TQ expressions) - dependent on first **B** mark only (this mark is for solving **their** 3TQ but not for solving $(3x+1)^2=0$)	Correct quadratic followed immediately by correct critical values (with no working) is **M0**
c.v. of x are $\frac{2}{9}, 4$	**B1**	**1.1**	Correct critical values of x (if factorisation shown then it must imply these two c.v.)	Must be $\frac{2}{9}$ or $0.\dot{2}$ but **B0** for $0.222\ldots$
$\left\{x : \frac{2}{9} < x < 4\right\}$	**B1FT dep***	**2.5**	FT their **two positive** critical values x_1, x_2 e.g. $\{x : x_1 < x < x_2\}$ where $x_2 > x_1$ allow $\left\{x : x > \frac{2}{9}\right\} \cap \{x : x < 4\}$ but 'union' is **B0** **B0** for interval notation e.g. $\left(\frac{2}{9}, 4\right)$	Answer must be in set notation for this mark – **dependent on first B mark only**
	[5]			

The mark scheme for question 2 is on the next page.

3.

DR				
$r(2x+3)=x+9$; $r(x+9)=2x-6$; $r^2(2x+3)=2x-6$	**B1**	**3.1a**	Obtain any correct equation in terms of r and x	Could be implied by later work May use other than r
$\frac{x+9}{2x+3}=\frac{2x-6}{x+9}$ oe	**M1***	**3.1a**	Attempt equation in terms of only x	This equation would imply the **B1** Or correct equation in terms of only r
$x^2+18x+81=4x^2-12x+6x-18$ $3x^2-24x-99=0$	**A1**	**1.1**	Obtain any correct equation not involving fractions or brackets	May still have like terms not yet combined May result in a cubic depending on method (probably $6x^3-39x^2-270x-297=0$)
$(x-11)(x+3)=0$ $x=11, x=-3$	**A1**	**2.1**	Solve quadratic **BC** to obtain both correct x values	Or solve cubic, to obtain three correct roots (third is likely to be $x=-1.5$)
$r=\frac{4}{5}$, $r=-2$	**A1**	**1.1**	Obtain at least $r=\frac{4}{5}$	If second value of r given then it must be correct (if third value given then it must be consistent with their correct cubic roots)
$S_\infty=\frac{a}{1-r}=\frac{25}{1-\frac{4}{5}}$	**M1d***	**3.2a**	Attempt sum to infinity, using correct formula, with their r and their attempt at a	Must be using their numerical values for a and r with $\lvert r\rvert<1$ **ISW** using additional value(s) of r **M0** if using their x and not attempt at a
$S_\infty=125$	**A1**	**1.1**	Obtain 125 only	**A0** if additional solution
S_∞ only exists for $\lvert r\rvert<1$, so $r=-2$ is not a valid solution	**B1**	**2.5**	Clear explanation as to why $r=-2$ is discarded	Must be considering correct r value, so **B0** if rejecting $x=-3$ as $\lvert-3\rvert>1$ Could generate the terms $-3, 6, (-12)$ and hence conclude with 'divergent sequence' If additional solutions for x and/or r from cubic then they must also be correct and explicitly rejected
	[8]			**NB** Eliminating x not r is a valid method, and could gain full credit. When solving their quadratic there is no need to see $r=-2$ (and hence $x=-3$) to award the **A** marks

2.

3(i)	$(5,-2)$ **or e.g.** $x=5, y=-2$ o.e.	B1	1.1b
		(1)	
(ii)	$(1.5,-2)$ **or e.g.** $x=1.5, y=-2$ o.e.	B1	1.1b
		(1)	
(iii)	$(-3, \ldots)$ **or** $(\ldots, -1)$ **or** $x=-3$ **or** $y=-1$ o.e.	B1	1.1b
	$(-3,-1)$ **or** $x=-3$ **and** $y=-1$ o.e.	B1	1.1b
		(2)	
			(4 marks)

4.

(a)		$(3-2x)^{-2}=\frac{1}{9}(1+\ldots)^{-2}$	**B1**	**1.1a**	For reference: $\frac{1}{9}\left(1-\frac{2}{3}x\right)^{-2}$ - soi	or for $3^{-2}(1+\ldots)^{-2}$
		$(1+kx)^{-2}=1+(-2)(kx)+\ldots$	**B1FT**	**1.1**	Correct first two terms follow through their k – allow un-simplified	$k\neq\pm1,\pm2$ - if correct $k=-\frac{2}{3}$
		$\ldots+\frac{(-2)(-3)}{2!}(kx)^2$	**B1FT**	**1.1**	Correct third term following through their k – allow un-simplified but must imply that the third term contains their k^2 - for correct k condone $\frac{2\times3}{2!}\left(\frac{2}{3}x\right)^2$ (or similar for their k if negative)	$k\neq\pm1,\pm2$ Condone $\frac{2\times3}{2!}(kx)^2$ and 2 for 2!
		$(3-2x)^{-2}=\frac{1}{9}\left(1+\frac{4}{3}x+\frac{4}{3}x^2+\ldots\right)$	**B1**	**1.1**	Or correct equivalent e.g. $\frac{1}{27}\left(3+4x+4x^2\right), \frac{1}{9}+\frac{4}{27}x+\frac{4}{27}x^2$, etc. ISW after correct expansion seen	Ignore higher order terms if found – a correct answer scores all 4 marks www
			[4]			
(b)		$\lvert x\rvert<\frac{3}{2}$	**B1**	**2.5**	oe, for example, $-\frac{3}{2}<x<\frac{3}{2}$ - allow $-\frac{3}{2}\leqslant x<\frac{3}{2}$ but not $-\frac{3}{2}\leqslant x\leqslant\frac{3}{2}$ (or any inequality that includes the $\frac{3}{2}$) - ISW once correct inequality seen. **Allow** $\left[-\frac{3}{2},\frac{3}{2}\right)$ or $\left(-\frac{3}{2},\frac{3}{2}\right)$ oe but not $\left[0,\frac{3}{2}\right)$ (or equivalents in set notation)	$-\frac{3}{2}<\lvert x\rvert<\frac{3}{2}$ is **B0** but $0\leqslant\lvert x\rvert<\frac{3}{2}$ is **B1** **Note that** $\lvert 2x\rvert<3$ **only is B0** (must be in terms of x)
			[1]			
(c)		$\frac{a+x}{(3-2x)^2}=(a+x)\left(\frac{1}{9}+\frac{4}{27}x+\ldots\right)$ $=\ldots+\left(\frac{1}{9}+\frac{4}{27}a\right)x+\ldots$	**B1FT**	**3.1a**	Finding correct coefficient of x **or** the x term for their $(p+qx+\ldots)(a+x)$ - **FT** their p and q from part **(a)** (so their x-coefficient must be $p+aq$). Allow embedded in an expansion e.g. $=\frac{1}{9}\left(\ldots+\left(\frac{4}{3}a+1\right)x+\ldots\right)$ or $=\frac{1}{9}\left(\ldots+\frac{4}{3}ax+x+\ldots\right)$	This mark can be implied by the correct answer for a (or on the **FT** as detailed in the next mark)
		$\frac{4}{3}a+1=0\Rightarrow a=-\frac{3}{4}$	**B1FT**	**2.2a**	Follow through $-\frac{\textit{their}\text{ constant term}}{\textit{their}\text{ coefficient of }x}$ from part **(a)**	
			[2]			

5.

11(a)	Explains that 3 is not in $3 < k < 4$ OE	2.4	E1	3 is not in the interval
	Subtotal		**1**	
11(b)(i)	Explains that $3 < y < x$ which contradicts the definition of x as the smallest value OE	2.4	E1	y is between 3 and x which contradicts the definition of x as the smallest value in (3, 4)
	Subtotal		**1**	
11(b)(ii)	Assumes there is a largest value in (3, 4) OE	2.1	B1	Step 1: Assume there is a largest number in the interval $3 < k < 4$ and let this largest number be x Step 2: let $y = \dfrac{x+4}{2}$ Step 3: $x < y < 4$ which is a contradiction. Step 4: Therefore, there is no largest value in $3 < k < 4$
	Constructs a value "y" in (3, 4) which is greater than their "x" Must have referenced their "x" before this step	2.2a	B1	
	States that $x < y < 4$ which is a contradiction OE	2.4	E1	
	Concludes that there is no largest value in (3, 4) OE CSO	2.1	R1	
	Subtotal		**4**	

6.

8(a)	$\text{fg}(2) = 4 - 3\left(\dfrac{5}{2(2)-9}\right)^2 = \ldots$	M1	1.1b
	$\text{fg}(2) = 1$	A1	1.1b
		(2)	
(b)	$y = \dfrac{5}{2x-9} \Rightarrow 2xy - 9y = 5 \Rightarrow 2xy = 5 + 9y$	M1	1.1b
	$2xy = 5 + 9y \Rightarrow x = \dfrac{5+9y}{2y}$	A1	2.1
	$g^{-1}(x) = \dfrac{5+9x}{2x} x \neq 0\{x \in \mathbb{R}\}$	A1	2.5
		(3)	
(c)(i)	$\{\text{gf}(x) =\}\ \dfrac{5}{2(4-3x^2)-9}$	M1	1.1b
	$= \dfrac{5}{-1-6x^2}$ or $\dfrac{-5}{1+6x^2}$	A1	1.1b
(ii)	$-5 \leqslant \text{gf}(x) < 0$	B1	2.2a
		(3)	

7.

(a)		$2\cot^2 x - 9\operatorname{cosec} x - 3[=0]$ $2\left(\frac{\cos^2 x}{\sin^2 x}\right) - 9\left(\frac{1}{\sin x}\right) - 3[=0]$	M1	2.1	Use of both $\cot x \equiv \frac{\cos x}{\sin x}$ and $\operatorname{cosec} x \equiv \frac{1}{\sin x}$	Condone use of *s* and *c* throughout but final answer must be in terms of sine – allow e.g. θ for *x* for **M** marks but must be *x* for the **A** mark
		$2\cos^2 x - 9\sin x - 3\sin^2 x[=0]$ $2(1-\sin^2 x) - 9\sin x - 3\sin^2 x[=0]$	M1	1.1	Correct use of $\sin^2 x + \cos^2 x \equiv 1$ to obtain an equation in $\sin x$ only	**Not dependent on the first M mark**
		$2 - 2\sin^2 x - 9\sin x - 3\sin^2 x[=0]$ $\Rightarrow 5\sin^2 x + 9\sin x - 2 = 0$	A1	2.2a	**AG** (so must be equal to zero) – sufficient working must be shown – any errors seen is **A0**	**A0** if an angle is missing from any trig. expression **used in their working**
			[3]			
(b)	(i)	**DR** $2\cot^2 2\theta - 9\operatorname{cosec} 2\theta - 3[=0]$ $\Rightarrow (5\sin 2\theta - 1)(\sin 2\theta + 2)[=0]$	M1	1.1	**SEE APPENDIX for awarding this mark (solving 3TQ expressions)**	condone using x for θ **or** 2θ for the **M** mark – condone for **M1** only $(5\sin\theta - 1)(\sin\theta + 2)$
		$\sin 2\theta = 0.2$ only as $\sin 2\theta \neq -2$	B1	2.3	Correctly stating that $\sin 2\theta = 0.2$ and that $\sin 2\theta$ cannot equal -2 (must explicitly reject the -2 (but no rationale required) - this mark is not implied by correct values for θ (as **DR** required)	Must be solving $5\sin^2 2\theta + 9\sin 2\theta - 2 = 0$ for the **B** marks condone $\sin 2x = 0.2$
		$[\theta =]\,0.101$	B1	1.1	awrt 0.101 (0.1006789…) www	
		$[\theta =]\,1.470$	B1	1.1	awrt 1.470 (1.4701173…) www	
					Ignore additional solutions outside of the range $0 < \theta < \pi$, but if any other solutions inside the range, award at most one of the two final **B** marks for one correct value	**SC B1** for awrt 0.10 **and** awrt 1.47 only if 3 dp or better) not seen **SC B1** for awrt 5.77 **and** awrt 84.2 only (working in degrees)
			[4]			
(b)	(ii)	$5\sin^2 2\theta + 9\sin 2\theta - 2[=0]$ $\Rightarrow 5(2\theta)^2 + 9(2\theta) - 2[=0]$ $\left(10\theta^2 + 9\theta - 1[=0]\right)$	M1	1.2	Use of the small angle approximation $\sin 2\theta \approx 2\theta$ twice in the given answer from **(a)** to obtain a three-term quadratic in θ (allow un-simplified)	Award **M1** only for $5\theta^2 + 9\theta - 2[=0]$ (so for using θ instead of 2θ) – allow e.g. *x* for θ
		$(10\theta - 1)(\theta + 1) = 0 \Rightarrow \theta = 0.10(000...)$ so is accurate to 2 decimal places	A1	2.4	State 0.10 (or better e.g. 0.100…) **as a decimal** following a correct quadratic in θ seen (no method required for solving the quadratic) and comment that this is accurate to 2 dp (**as a minimum** must mention '2 dp' with the value of 0.10(000…) appearing in this part and 0.10 **or** 0.101 **or** 0.100(6789…) appearing in part **(b)(i)**)	**This mark is dependent on an awrt 0.10 seen in part (b)(i) or a correct sign change test (see below)** **Ignore any consideration of other root(s)** **SEE APPENDIX FOR ALTERNATIVE**
			[2]			

8.

(a)		common difference of B is $-d$ $(b_1 = a_{100} =)\ 4 + 99d$ $b_i = 4 + 99d + (i-1)(-d)$ or $4 + 100d - id$ $4 + 100d - id = 4 + (i-1)d + 10$ $(2id = 101d - 10)$ $\Rightarrow i = \frac{101}{2} - \frac{5}{d}$ **AG**	B1 B1 M1 A1 M1 A1 [6]	1.1 1.1 3.1a 2.1 1.1 1.1	soi soi (or $a_1 + 99d$) For an attempt at the general term b_i using b_1 ($= a_{100}$), i.e. an expression of the form $a_1 + kd + (i-1)(\pm d)$ oe Fully correct expression for b_i Equating their b_i to an expression for a_i +10 May see $a_1 + (i-1)d + 10$ www (so this mark is dependent on all previous marks) May see a_1 used throughout instead of 4 Condone other letters used throughout (e.g. n for i and c for $-d$): - Must recover to d in order to solve (A1A1) - Must recover to i to demonstrate the AG (A1)
(b)		$\frac{101}{2} - \frac{5}{d}$ not always integer [between 1 & 100] $d = 2$ or 10	B1 B1 [2]	2.4 3.2b	soi (accept e.g. "i must be an integer") Ignore negative values (but B0 for any additional positive values)

The mark scheme for Qu 9 is on the next page

10.

(a)	(i)	**DR** $\cos 3\theta = \cos(2\theta + \theta)$ $= (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta\sin\theta$ $= 2\cos^3\theta - \cos\theta - 2(1 - \cos^2\theta)\cos\theta$ $= 4\cos^3\theta - 3\cos\theta$ **AG**	 B1 B1	 1.1 1.1	 Condone a small slip in early irrelevant working before substituting, but must be fully correct after this. oe in terms of θ, not 2θ May be implied by next line oe in terms of $\cos\theta$ only Must reach AG or give a conclusion (even if just 'QED') www
		Alternative 1 $= (\cos^2\theta - \sin^2\theta)\cos\theta - 2\sin\theta\cos\theta\sin\theta$ $= \cos^3\theta - (1 - \cos^2\theta)\cos\theta - 2(1 - \cos^2\theta)\cos\theta$ $= 4\cos^3\theta - 3\cos\theta$ **AG**	 B1 B1		 oe in terms of θ, not 2θ May be implied by next line oe in terms of $\cos\theta$ only Must reach AG or give a conclusion (even if just 'QED') www
		Alternative 2 $= (1 - 2\sin^2\theta)\cos\theta - 2\sin\theta\cos\theta\sin\theta$ $= (1 - 2(1 - \cos^2\theta))\cos\theta - 2(1 - \cos^2\theta)\cos\theta$ $= 4\cos^3\theta - 3\cos\theta$ **AG**	 B1 B1 [2]		 oe in terms of θ, not 2θ May be implied by next line oe in terms of $\cos\theta$ only Must reach AG or give a conclusion (even if just 'QED') www
(a)	(ii)	**DR** $\cos 3\theta = \frac{\sqrt{2}}{2}$ $3\theta = 45°$ or $315°$ or $405°$ $\theta = 15°$ or $105°$ or $135°$	 M1 A1 A1 [3]	 1.1 1.1 1.1	 oe Allow A1 for two correct values of 3θ. Ignore other values. This mark is not implied by correct final answers but accept equivalent correct working e.g. a graph of $\cos 3\theta$. Accept radians for this mark only: $3\theta = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}$ cao. Ignore values outside of the range 0-180°, but do not accept radians for this mark. This mark may be given following M1A0.
(b)	(i)	**DR** $4x^3 - 2\sqrt{2}x^2 - x + 2\sqrt{2}x^2 - 2x - \frac{\sqrt{2}}{2}$ oe $= 4x^3 - 3x - \frac{\sqrt{2}}{2}$ **AG**	 B1 [1]	 1.1	 Must see a correct multiplied out form and AG or conclusion.
(b)	(ii)	**DR** $x = -\frac{\sqrt{2}}{2}$ $x = \frac{\sqrt{2}+\sqrt{6}}{4}$ and $x = \frac{\sqrt{2}-\sqrt{6}}{4}$ oe	 B1 B1 [2]	 2.1 1.1	 oe, must be exact May see $x = \frac{2\sqrt{2}\pm\sqrt{24}}{8}$ etc.
(c)		**DR** $\cos 15°$ is a root of the equation in **(b)(ii)** **(a)(ii):** $\cos 105° < 0$, $\cos 135° < 0$, $\cos 15° > 0$ **(b)(ii):** $-\frac{\sqrt{2}}{2} < 0$ and $\frac{\sqrt{2}-\sqrt{6}}{4} < 0, \frac{\sqrt{2}+\sqrt{6}}{4} > 0$ $\cos 15° = \frac{\sqrt{2}+\sqrt{6}}{4}$ **AG**	 B1 B1 [2]	 2.4 3.2a	 soi - for 'spotting' the connection. This mark can be gained regardless of their answers to **(a)(ii)** and **(b)(ii)**. $x = \cos\theta$ or 3 correctly paired roots are sufficient for this mark Condone $x = \cos 15$ Justification for selecting this root (may say e.g. "cos 15 is the **only** positive root") This may be implied by matching each pair of answers correctly (but all three must be present or mentioned).

9.

$\frac{1}{2}x-1=2x-3$ $x=\frac{4}{3}$	**B1**	**1.1**	Obtain $x=\frac{4}{3}$ oe	
$\frac{1}{2}x-1=-2x+3$	**M1**	**1.1**	Attempt to solve equation with all signs reversed on one side of the equation, or square both sides and attempt to solve	**M0** for eg $\frac{1}{2}x-1=-2x-3$
$x=\frac{8}{5}$	**A1**	**1.1**	Obtain $x=\frac{8}{5}$ oe	Maximum of 2 marks if additional solutions
	[3]			
Alternative method				
$\left(\frac{1}{2}x-1\right)^2=(2x-3)^2$ $\frac{1}{4}x^2-x+1=4x^2-12x+9$ $15x^2-44x+32=0$ $(3x-4)(5x-8)=0$	**M1**		Square both sides to obtain two 3 term quadratics, and attempt to solve	Possibly **BC**
$x=\frac{4}{3}$	**A1**		Obtain $x=\frac{4}{3}$	
$x=\frac{8}{5}$	**A1**		Obtain $x=\frac{8}{5}$	Maximum of 2 marks if additional solutions

11.

(a)	A, C <u>or</u> A, D <u>or</u> B, D [Allow things like $A \cap D$]	B1 **(1)**	1.2
(b)	$P(C) = 0.6$ <u>and</u> $P(B) = p + 0.32$ <u>and</u> $P(B \cap C) = 0.27$ <u>or</u> $(0.08+0.25+0.27)\times(0.27+0.05+p)=0.27$ <u>or</u> $0.27+0.05+p=\frac{0.27}{0.6}=0.45$	M1	1.1b
	$[p + 0.32 = 0.45$ so$]$ $p =$ **<u>0.13</u>**	A1 **(2)**	2.2a
(c)	$[P(A \mid B')] = \frac{q}{q+r+0.25+0.08}$ <u>or</u> $\frac{q}{1-(0.05+"0.13"+0.27)}$ <u>or</u> $\frac{q}{0.55}$	M1	2.1
	$q + r = 1 - 0.65 -$ "0.13" $[= 0.22]$	M1	1.1b
	Since $r \geqslant 0$ the greatest value of q is "0.22" so $P(A \mid B') \leqslant$ **<u>0.4</u>** or $\underline{\frac{2}{5}}$	A1 **(3)**	2.2a
(d)	$[P(B \mid A')=]\frac{0.27+"0.13"}{0.6+"0.13"+r}=0.5$ or $\frac{0.27+"0.13"}{1-(q+0.05)}=0.5$	M1	1.1b
	$r =$ **<u>0.07</u>** ,	A1	1.1b
	$q =$ **<u>0.15</u>**	A1ft **(3)**	1.1b