Oxford Cambridge and RSA

Practice Paper 1

A Level Mathematics A

H240/01 Pure Mathematics

MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 100

Version Final

Last updated 07/12/17

This document consists of 11 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(i)		**DR** $\sqrt{36}+\sqrt{162}$ oe $\sqrt{6^2}+\sqrt{9^2\times 2}$ oe $= 6 + 9\sqrt{2}$	**M1** **A1** **A1** **[3]**	**1.1a** **1.1** **1.1**	Attempt to expand bracket Obtain 6 Obtain $9\sqrt{2}$	 Must show sufficient method
	(ii)		**DR** $\frac{6(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})}=\frac{12-6\sqrt{2}}{2}=6-3\sqrt{2}$	**M1** **A1** **A1** **[3]**	**1.1a** **1.1** **1.1**	Multiply numerator and denominator by $2-\sqrt{2}$ Either numerator or denominator correct Fully correct expression	 Must be simplified Must show sufficient method
2			**DR** $3(1-\cos^2\theta)-2\cos\theta-2=0$ $3\cos^2\theta+2\cos\theta-1=0$ $(3\cos\theta-1)(\cos\theta+1)=0$ $\cos\theta=\frac{1}{3}$ $\cos\theta=-1$ $\theta=70.5^\circ, 289^\circ, 180^\circ$	**M1** **A1** **M1** **A1** **A1** **[5]**	**3.1a** **1.1** **1.1a** **2.2a** **1.2**	Attempt to use $\sin^2\theta = 1-\cos^2\theta$ Obtain correct equation Attempt to solve quadratic Obtain at least two correct angles Obtain all 3 angles, and no others	 Factorise or **BC**
3	(i)		$2x^2+4x+5=2(x^2+2x)+5$ $=2[(x+1)^2-1]+5$ $=2(x+1)^2+3$	**B1** **B1** **M1** **A1** **[4]**	**2.2a** **1.1** **1.1a** **1.1**	$p = 2$ $q = 1$ Attempt r $r = 3$	The values of p, q and r could be stated explicitly or could be implied by an answer in completed square form
	(ii)		Vertex is at (-1, 3)	**B1ft** **B1ft** **[2]**	**1.1** **1.1**	Correct x coordinate Correct y coordinate	FT their **(i)**

Question			Answer	Marks	AO	Guidance	
	(iii)		$k < 3$	**B1ft** **[1]**	**3.1a**	State $k < 3$, ft their **(i)**	Must be strict inequality
4	**(i)**		$V = x(21 - 2x)^2$ $= 4x^3 - 84x^2 + 441x$ $V' = 12x^2 - 168x + 441$ $12x^2 - 168x + 441 = 0$ $x = 3.5$ cm when $x = 3.5$, $V'' = 24 \times 3.5 - 160 < 0$ hence maximum	**B1** **M1** **M1** **A1ft** **M1** **A1ft** **[6]**	**3.3** **1.1a** **3.1b** **3.2a** **1.1** **2.1**	Sate correct expression for volume Expand and attempt differentiation Equate to 0 and attempt to solve Obtain $x = 3.5$ cm only, ft on their V Use second derivative oe Conclude maximum	oe Or use product rule **BC** A0 if 10.5 also given If evaluated, must be correct Evidence required
	(ii)		Accept any sensible assumption	**B1** **[1]**	**3.5b**	E.g. Thickness of metal is assumed negligible	
5	**(i)**		$f(x) = (x + 1)(x^2 + 3x - 10)$ $= (x + 1)(x + 5)(x - 2)$	**M1** **A1** **A1** **[3]**	**2.2a** **1.1** **1.1**	Attempt complete division by $(x + 1)$ Obtain correct quotient Obtain fully factorised $f(x)$	Allow any equiv method Must be as product
	(ii)		$e^y = -1, -5, 2$ but $e^y > 0$, so $e^y = 2$ is only valid root hence $y = \ln 2$	**M1** **E1** **A1** **[3]**	**2.2a** **2.4** **2.1**	Link e^y to attempt at roots from **(i)** Explanation that $e^y > 0$ Obtain $y = \ln 2$	 www
6	**(i)**		Points at (30, 1.53), (40, 1.62), (50, 1.70)	**B1** **[1]**	**1.1**	Plot $\log_{10}P$ against t	Allow one error
	(ii)		$\log_{10}a = 1.30$ so $a = 20$ $\log_{10}b = 0.008$ $b = 1.02$	**B1** **M1** **A1** **[3]**	**3.3** **3.4** **1.1**	Correct value for a State or imply that gradient is $\log_{10}b$ Obtain $b = 1.02$ (awrt)	Could just be stated Method must show use of graph not substitution into given model

Question			Answer	Marks	AO	Guidance	
	(iii)		Answer in range 700 to 1050	**B1ft** **[1]**	**3.4**	ft their *a* and *b*	
	(iv)		Accept any sensible explanation	**B1** **[1]**	**3.5b**	Eg extrapolation unreliable Eg the model is continuous, not discrete	Eg Model may no longer be valid eg insufficient food to support larger population
7			$u = \ln x$, $\mathrm{d}v = 2x + 1$	**M1**	**1.1a**	Recognise integration by parts with correct *u* and d*v*	
			$\mathrm{d}u = \frac{1}{x}$, $v = x^2 + x$	**B1**	**1.2**	State or imply that $\mathrm{d}u = \frac{1}{x}$	
			$I = (x^2 + x)\ln x - \int (x^2 + x)\frac{1}{x}\,\mathrm{d}x$	**A1**	**1.1**	Correct unsimplified expression	
			$= (x^2 + x)\ln x - \int (x+1)\,\mathrm{d}x$	**M1**	**1.1a**	Attempt to simplify and integrate	
			$= (x^2 + x)\ln x - (\frac{1}{2}x^2 + x) + c$	**A1** **[5]**	**1.1**	Obtain fully correct integral	Including + *c*
8	**(i)**		(0, 4]	**B1** **[1]**	**2.5**	Do not allow $0 < f(x) \leq 4$	
	(ii)		$f^{-1}(x) = \dfrac{8}{x} - 2$	**M1**	**1.1a**	Obtain $\dfrac{8}{x} \pm 2$	Allow in terms of *y*
				A1 **[2]**	**1.1**	Obtain correct inverse function	Must now be in terms of *x*
	(iii)		$x = \dfrac{8}{x+2}$ $x^2 + 2x - 8 = 0$	**M1**	**1.1a**	Equate two of *x*, f(*x*) and $f^{-1}(x)$ and attempt to solve	
			$x = 2$	**A1** **[2]**	**2.3**	Obtain *x* = 2 only	A0 if *x* = –4 also given

Question			Answer	Marks	AO	Guidance	
9	**(i)**	**(a)**	$0.5\times1.5\times\{8.0+8.6+2(8.5+8.2+8.40)\}=50.1$	**M1** **A1** **[2]**	**1.1a** **1.1**	Attempt use of correct trapezium rule Obtain correct area	
	(i)	**(b)**	50.1×0.49 = £24.55	**B1ft** **[1]**	**1.1**	Obtain cost of £24.55, ft their area	
	(ii)		Could be under-estimate as modeling tops of trapezia with straight lines Lawn seed may only be sold in fixed volumes so may not be able to buy exact volume needed	**B1** **B1** **[2]**	**3.5b** **3.5b**	Limitation based on use of trapezium rule Limitation based on buying lawn seed	Accept any two sensible comments
	(iii)		Any sensible refinement	**B1** **[1]**	**3.5c**	Eg Use trapezium rule with more strips	
	(iv)		$2\sin^{-1}(0.9375)=2.43$ rads $0.5\times3.2^2\times(2.43-\sin2.43)=9.10$ 9.10×0.17 = £1.55	**M1** **A1** **M1** **A1** **B1ft** **[5]**	**3.1b** **1.1** **1.1a** **1.1** **3.2a**	Attempt to find angle (rads or degs) Obtain 2.43 rads, or 139^o Attempt complete method to find area of segment Obtain 9.10 (m^2) Obtain cost of £1.55, ft their area	Could use cosine rule Allow 1.22 rads or 69.6^o Allow 9.1 Must include units

Question			Answer	Marks	AO	Guidance	
10			$(5t + 3) + 4(n - 1) = (17t + 11)$	M1	3.1a	Attempt to use $a + (n - 1)d = l$	
			$n = 3t + 3$	A1	2.1	Obtain $n = 3t + 3$	
			$S_N = \frac{1}{2}(3t + 3)\{(5t + 3) + (17t + 11)\}$	M1	2.1	Attempt to find sum of AP	
			$S_N = \frac{1}{2}(3t + 3)(22t + 14) = 3(t + 1)(11t + 7)$	A1	2.1	Obtain $S_N = 3(t + 1)(11t + 7)$ oe	
			When t is odd, $t = 2k + 1$ so	E1	2.2a	Consider S_N when t is odd	Allow consideration of odd and even factors
			$S_N = 3(2k + 2)(22t + 18)$ $= 12(k + 1)(11k + 9)$ hence multiple of 12	E1	2.4	Fully correct and convincing proof	
			When t is even, $t = 2k$ so $S_N = 3(2k + 1)(22k + 7)$ hence always odd	E1	2.4	Allow worded eg $3 \times$ odd $\times$ odd	
				[7]			
11	(i)		$1 + x - \frac{1}{2}x^2$	B1	1.1	Obtain $1 + x$	Terms must be simplified for B / A marks
				M1	1.1a	Attempt third term	
				A1	1.1	Obtain correct third term	
				[3]			
	(ii)		$\sqrt{1.08} \approx 1 + 0.04 - 0.5 \times 0.04^2$	M1	2.1	Substitute 0.04 throughout	Need $\sqrt{1.08}$ as well
			$\sqrt{0.36 \times 3} \approx 1.0392$ $0.6\sqrt{3} \approx 1.0392$	M1	3.1a	Rearrange $\sqrt{1.08}$ to $k\sqrt{3}$	
			$\sqrt{3} \approx 1.73$	A1	1.1	Obtain 1.73 or better	Must see method
				[3]			
	(iii)		Expansion is only valid for $\lvert x \rvert < \frac{1}{2}$	E1	2.3	Explanation must be specific	
				[1]			

Question			Answer	Marks	AO	Guidance	
12			**DR**				
			$\sin y + x\cos y\dfrac{dy}{dx} - 2\sin 2y\dfrac{dy}{dx} = 0$	**B1**	**1.1a**	Correct derivatives of cos*y* and – 2sin2*y*	
				M1	**1.1**	Attempt use of product rule for *x*sin*y*	
				A1	**1.1**	Obtain correct derivative	
			$\dfrac{dy}{dx} = \dfrac{\sin y}{2\sin 2y - x\cos y}$				
			$2\sin 2y - x\cos y = 0$	**M1**	**3.1a**	Rearrange and use denominator = 0	
			$4\sin y\cos y - x\cos y = 0$	**M1**	**3.1a**	Use $\sin 2y = 2\sin y\cos y$ and attempt solution	
			$\cos y(4\sin y - x) = 0$ so $\cos y = 0$ or $x = 4\sin y$				
			$\cos y = 0$ gives $(\frac{7}{2}, \frac{1}{2}\pi)$	**A1**	**2.1**	Obtain $(\frac{7}{2}, \frac{1}{2}\pi)$	
			$x = 4\sin y$ gives $4\sin^2 y + \cos 2y = 2.5$ $4\sin^2 y + 1 - 2\sin^2 y = 2.5$ $\sin y = \pm\frac{1}{2}\sqrt{3}$	**M1**	**3.1a**	Substitute $x = 4\sin y$ into original equation and attempt to solve	Including use of correct identity
			$\sin y = \frac{1}{2}\sqrt{3}$ gives $(2\sqrt{3}, \frac{1}{3}\pi)$ and $(2\sqrt{3}, \frac{2}{3}\pi)$	**A1**	**3.2a**	Obtain one correct solution	
			$\sin y = -\frac{1}{2}\sqrt{3}$ gives $x < 0$, so no valid solutions	**A1**	**2.4**	Obtain both correct roots	Must discount $\sin y = -\frac{1}{2}\sqrt{3}$
				[9]			
13	(i)		1.4422, 1.5099, 1.5197, 1.5211, 1.5213, 1.5214...	**B1**	**1.1a**	Correct x_2	
				M1	**1.1**	Use correct iterative process	
			Hence $\alpha = 1.521$	**A1**	**2.2a**	Obtain 1.521 (must be 4sf)	
				[3]			
	(ii)		$F'(x) = -(x^2 + 4x)x^{-4}$	**B1**	**1.1**	Correct $F'(x)$	
			$F'(\alpha) = -1.57$	**B1ft**	**1.1**	Correct $F'(\alpha)$	Follow their value of α
			Will only converge if $\lvert F'(\alpha)\rvert < 1$	**E1**	**1.2**	Identify correct condition	
				[3]			

Question			Answer	Marks	AO	Guidance	
14	(i)	(a)	when $x = 0$, $t = 0$ and hence $y = 0$	**E1** **[1]**	**2.4**	Justify (0, 0) convincingly	
		(b)	when $x = 1$, $t = 1$ and hence $y = 0.5$	**B1** **[1]**	**1.1**	Obtain $y = 0.5$	
	(ii)		$\frac{dx}{dt} = \frac{2}{(1+t)^2}$	**M1**	**2.1**	Attempt $\frac{dx}{dt}$	Using quotient rule, or other valid method
				A1	**2.1**	Obtain correct derivative	
			$\int \frac{t^2}{1+t} dx = \int \frac{t^2}{1+t} \times \frac{2}{(1+t)^2} dt$	**M1**	**2.1**	Use $\int y dx = \int y \frac{dx}{dt} dt$	
			$= \int \frac{2t^2}{(1+t)^3} dt$	**A1**	**2.1**	Obtain given answer	
				B1 **[5]**	**2.4**	Justify t-limits from $x = 0, 1$	$x = 0$: $\frac{2t}{1+t} = 0$ so $t = 0$ $x = 1$: $\frac{2t}{1+t} = 1$ $2t = 1 + t$ so $t = 1$
	(iii)		**DR**				
			use $u = 1 + t$ giving $du = dt$	**E1**	**1.1a**	Must be stated explicitly	
			$\int \frac{2t^2}{(1+t)^3} dt = \int \frac{2(u-1)^2}{u^3} du$	**M1**	**1.1a**	Attempt to change integrand to function of u	
			$= \int 2u^{-1} - 4u^{-2} + 2u^{-3} du$	**A1**	**1.1**	Obtain correct integrand	Any equivalent form
			$= \left[2\ln u + 4u^{-1} - u^{-2}\right]_1^2$	**M1**	**1.1a**	Attempt integration	
			$= (2\ln2 + 2 - 0.25) - (2\ln1 + 4 - 1)$	**M1**	**1.1a**	Attempt use of limits $u = 1, 2$	
			$= 2\ln2 - \frac{5}{4}$	**A1** **[6]**	**1.1**	Obtain correct exact area	Allow any exact equiv