OCR
Oxford Cambridge and RSA

Practice Paper – Set 3

A Level Mathematics A
H240/01 Pure Mathematics

MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 100

FINAL

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(i)		$-6 < 3x < 13$	M1	1.1	Attempt to solve two equations / inequalities each involving all three terms	Correct order of operations
			$-2 < x < \frac{13}{3}$	A1	1.1	Obtain correct inequality	
				[2]			
	(ii)		$4x^2 > 25$	M1	1.1	Rearrange to useable form	Or $4x^2 - 25 > 0$
			$-\frac{5}{2}, \frac{5}{2}$	M1	1.1	Attempt to find critical values	or **BC**
			$x < -\frac{5}{2}$ or $x > \frac{5}{2}$	A1FT	2.2a	Choose 'outside' region for inequality FT their critical values	or **BC**
				[3]			
2	(i)		$a = 0.6$	B1	1.2	State correct value for a	
				[1]			
	(ii)		$3k = 0.6$, so $k = 0.2$	M1	1.1a	Attempt to find scale factor	**OR** $0.6k = 3$, so $k = 5$
			$a = 8 \times 0.2 = 1.6$	A1	1.1	Obtain $a = 1.6$	
				[2]			
	(iii)		$\sqrt{a^2 + 0.6^2} = 1$	B1	1.2	Correct definition for unit vector seen or implied	Allow BOD for $a^2 + 0.6^2 = 1$, with no square root seen
			$a^2 = 0.64$	M1	1.1a	Attempt to find at least one value for a	
			$a = \pm 0.8$	A1	1.1	Both correct values for a	
				[3]			
3	(i)		$(\frac{2}{3}, 0)$, $(0, 2)$, $(-\frac{1}{2}, 0)$ and $(0, 1)$ all indicated	M1	1.2	one of $(\frac{2}{3}, 0)$ or $(-\frac{1}{2}, 0)$	
				M1	1.1	one of $(0, 2)$ or $(0, 1)$	
				A1	1.1	All four	
				[3]			
	(ii)		$2x + 1 = 3x - 2$, so $x = 3$	B1	1.1	Obtain $x = 3$ www	
			$2x + 1 = -3x + 2$	M1	1.1	Attempt to solve equation, with $2x$ and $3x$ of opposite signs	or by squaring both sides
			$x = \frac{1}{5}$	A1	1.1	Obtain $x = \frac{1}{5}$ www	
				[3]			

Question			Answer	Marks	AO	Guidance	
4	(i)		$0.5\times10\times7\times\sin0.8=25.107$	M1	3.1a	Attempt area of triangle	
			$0.5\times r^2\times0.8=12.55$	M1	1.1	Equate area of sector to half of area of triangle and attempt to find r	
			r = 5.6 cm	A1 [3]	1.1	Obtain correct value for r	5.6 or better (5.60217…)
	(ii)		$CD = 5.6\times0.8 = 4.48$	M1	3.1a	Attempt arc length using $r\theta$	
			$AB=\sqrt{7^2+10^2-2\times7\times10\times\cos0.8}=7.17$	M1	1.1	Attempt AB using cosine rule	
			$ABCD$ = 7.17 + (10 – 5.6) + 4.48 + (7 – 5.6)	M1	1.1	Attempt perimeter of $ABCD$	
			= 17.5 cm	A1 [4]	1.1	Obtain 17.5 cm	Allow 17.4 www
5	(i)		E.g. $\log_3x^2 = 2\log_3x$; the student has ignored the brackets and used the power rule incorrectly	E1	2.3	Error identified with explanation	
			E.g. $x = 3^2$; the student has done 2^3	E1 [2]	2.3	Error identified with explanation	
	(ii)		$(2\log_3x + 1)(\log_3x – 2) = 0$	M1	3.1a	Attempt to solve quadratic in $\log_3x$	soi
			$\log_3x = – 0.5$, $\log_3x = 2$	A1	1.1	Obtain two correct roots **BC**	
			$x = 3^{-0.5}$ or $x = 3^2$	M1	1.1a	Attempt correct process to find x at least once	
			$x = \frac{1}{3}\sqrt{3}$ and $x = 9$	A1 [4]	1.1	Obtain both correct roots	Any equivalent exact form

Question			Answer	Marks	AO	Guidance	
6			**DR**				
			$\frac{2(3x-1)-6x}{(3x-1)^2}$	**M1**	**1.1**	Attempt to differentiate first term using quotient rule or equiv	
			$\frac{5}{2}(5x+1)^{-\frac{1}{2}}$	**M1**	**1.1**	Attempt to differentiate second term	Obtain $k(5x+1)^{-\frac{1}{2}}$
			$\frac{dy}{dx}=\frac{-2}{(3x-1)^2}+\frac{5}{2}(5x+1)^{-\frac{1}{2}}$	**A1**	**2.1**	Fully correct derivative	Allow unsimplified
			at $x = 3$, $\frac{dy}{dx}=-\frac{2}{64}+\frac{5}{8}=\frac{19}{32}$	**M1**	**1.1a**	Attempt gradient at $x = 3$	
			at $x = 3$, $y = \frac{19}{4}$	**B1**	**1.1**	Correct y-coordinate	
			$y-\frac{19}{4}=\frac{19}{32}(x-3)$	**M1**	**2.1**	Attempt equation of line with their y-coordinate and gradient	
			$32y - 152 = 19x - 57$ $19x - 32y + 95 = 0$ **AG**	**A1**	**2.1**	Rearrange to given form	At least one line of working seen
				[7]			

Question			Answer	Marks	AO	Guidance	
7	(i)		$x^2 + 4x^2 + 2x - 32x + 56 = 0$ $5x^2 - 30x + 56 = 0$	**M1**	**2.1**	Substitute $y = 2x$ into equation of circle and rearrange to three term quadratic	
			$30^2 - 4 \times 5 \times 56 = -220$	**M1**	**2.4**	Consider discriminant	
			$b^2 - 4ac < 0$ hence no real roots so the circle and line do not intersect	**A1** **[3]**	**2.2a**	Conclude with no real roots	
	(ii)	(a)	Centre of circle is (-1, 8)	**B1**	**1.1**	Seen or used	
			Gradient of perpendicular is -0.5	**B1**	**2.2a**	For gradient of perpendicular	
			$y - 8 = -0.5(x + 1)$	**M1**	**1.1**	Attempt equation of line through their circle centre with gradient of -0.5	
			$x + 2y = 15$	**A1** **[4]**	**1.1**	Obtain correct equation	Allow any 3 term equivalent
		(b)	$5x = 15$ $x = 3$, $y = 6$	**M1**	**3.1b**	Attempt to solve simultaneously with $y = 2x$	
			distance from centre to line is $\sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$	**M1**	**1.1**	Use Pythagoras to find distance between centre of circle and point of intersection	
			$(x + 1)^2 + (y - 8)^2 = 3^2$	**M1**	**1.1a**	Attempt to find radius of circle	Seen at any point in solution – allow back credit to part (a) if the radius is found at the same time as the centre of circle
			hence shortest distance between line and circle is $2\sqrt{5} - 3$	**A1** **[4]**	**3.2a**	Obtain $2\sqrt{5} - 3$	Allow any exact equiv

Question			Answer	Marks	AO	Guidance	
8	(i)		$\frac{2}{3}\pi r^3 + \pi r^2 h = 45\pi$	B1	3.1b	Equate correct volume to 45π	
			$A = \pi r^2 + 2\pi r^2 + 2\pi rh$	B1	1.1	Correct expression for surface area	
			$h = \dfrac{45 - \frac{2}{3}r^3}{r^2} = 45r^{-2} - \frac{2}{3}r$	M1	1.1	Attempt to make h the subject and hence eliminate h	
			$A = 3\pi r^2 + 2\pi r(45r^{-2} - \frac{2}{3}r)$ $= 3\pi r^2 + 90\pi r^{-1} - \frac{4}{3}\pi r^2$				
			$A = \frac{5}{3}\pi r^2 + \dfrac{90\pi}{r}$ **AG**	A1	2.1	Simplify to obtain given answer	
				[4]			
	(ii)		$\dfrac{dA}{dr} = \frac{10}{3}\pi r - 90\pi r^{-2}$	M1	1.1a	Attempt differentiation	
			$\frac{10}{3}\pi r - 90\pi r^{-2} = 0 \quad \Rightarrow \quad r = 3$	M1	3.1b	Equate to 0 and solve for r	
			$A = 45\pi$ cm^2 or 141 cm^2	A1	3.2a	Correct surface area, including units	
			$\dfrac{d^2A}{dr^2} = \frac{10}{3}\pi + 180\pi r^{-3} > 0$ hence minimum	A1FT	2.2a	FT their first derivative, provided it gives a minimum	Or using the sign-change of first derivative
				[4]			
	(iii)		E.g. Cheaper to manufacture as uses less material	E1	3.2b	Sensible reason based on surface area	
				[1]			

Question			Answer	Marks	AO	Guidance	
9	(i)		$S = Ae^{kt}$	B1	3.3	State or imply appropriate exponential model	Other models are possible eg using t as number of years after a year other than 2015 OR $S = ab^t$
			$S = 3.1e^{kt}$	B1	3.3	Identify correct initial value	OR $a = 3.1$
			$\frac{dS}{dt} = 3.1ke^{kt}$	M1	3.3	Attempt differentiation	May still be A and not 3.1 OR $\frac{dS}{dt} = 3.1(\ln b)b^t$
			$0.8 = 3.1ke^0$ hence $k = 0.258$	M1	3.4	Substitute into derivative and attempt to find k	OR $0.8 = 3.1(\ln b)$ so $b = 1.29$
			$S = 3.1e^{0.258t}$, where S is the annual sales in millions of devices and t is the number of years after 2015	A1 [5]	2.5	Correct equation with variables clearly defined	OR $S = 3.1(1.29)^t$
	(ii)		when $t = 2$, $S = 3.1e^{0.516} = 5.19$ (millions)	M1	3.4	Find value of S when $t = 2$	Using their model which must be of the form Ae^{kt} or ab^t, with numerical parameters
			E.g. so observed value was 5.2 (millions) so model appears to be reliable	E1 [2]	3.5a	Comment on reliability of model	Must have correct 5.2 million, from correct model
	(iii)		E.g. unlikely to be a reliable prediction as market will become saturated so sales unlikely to increase at same rate	E1 [1]	3.5b	Comment about trend unlikely to continue, or device becoming obsolete or extrapolation may not be reliable	

Question			Answer	Marks	AO	Guidance	
10	(i)	(a)	$f'(x) = \frac{3}{3x+1} - 1$	B1	1.1	Correct $f'(x)$	
			$\frac{3}{3x+1} - 1 = 0$	M1	2.1	Equate to 0 and solve for x	Must be seen
			$x = \frac{2}{3}$	A1	1.1	Obtain $x = \frac{2}{3}$	
				[3]			
		(b)	E.g. The tangent at α will be parallel to the x-axis so will never intersect the x-axis to give x_2.	E1	2.4	Correct reason referring to the tangent at α	A sketch is allowable if annotated.
				[1]			
	(ii)		$x_{n+1} = x_n - \frac{\ln(3x_n+1) - x_n}{\frac{3}{3x_n+1} - 1}$	B1	2.1	State correct N-R formula	Allow no subscripts
			$x_{n+1} = x_n - \frac{(3x_n+1)(\ln(3x_n+1) - x_n)}{3-(3x_n+1)}$	M1	2.1	Attempt rearrangement	
			$x_{n+1} = \frac{x_n(2-3x_n) - (3x_n+1)\ln(3x_n+1) + x_n(3x_n+1)}{2-3x_n}$				
			$x_{n+1} = \frac{2x_n - 3x_n^2 - (3x_n+1)\ln(3x_n+1) + 3x_n^2 + x_n}{2-3x_n}$	A1	2.5	Obtain given N-R formula	
			$x_{n+1} = \frac{3x_n - (3x_n+1)\ln(3x_n+1)}{2-3x_n}$ **AG**				
				[3]			
	(iii)		$x_2 = 2.54518$	B1	1.1a	Correct first iterate	At least 4sf
			$x_3 = 1.94865$, $x_4 = 1.90416$,	M1	1.1a	Use correct iterative process to find at least two further values	At least 4sf
			$x_5 = 1.90381$, $x_6 = 1.90381$				
			$\beta = 1.9038$	A1	1.1	Obtain $\beta = 1.9038$	From at least 5 iterates, to at least 5sf
				[3]			

Question			Answer	Marks	AO	Guidance	
	(iv)		$\ln(3\times 1.90375 + 1) - 1.90375 = 0.000035223$ $\ln(3\times 1.90385 + 1) - 1.90385 = -0.000020077$ f(1.90375) > 0 and f(1.90385) < 0 so by sign change $1.90375 < \beta < 1.90385$ so must 1.9038 correct to 5sf **[3]**	**M1** **A1*** **dep*A1**	**2.1** **2.1** **2.4**	Attempt values either side of root Obtain both correct values Must refer to sign change	Allow other valid methods

Question			Answer	Marks	AO	Guidance	
11			**DR** $a + d = ar^2$	**B1**	**3.1a**	Correct equation for $a_2 = g_3$	Allow a, a_1, g_1 or $1+\sqrt{5}$ and may be different in each term eg $a_1 + d = g_1r^2$
			$a + 2d + ar^3 = 0$	**B1**	**3.1a**	Correct equation for $a_3 + g_4 = 0$	Allow a, a_1, g_1 or $1+\sqrt{5}$ and may be different in each term eg $a_1 + 2d + g_1r^3 = 0$
			$a + 2(ar^2 - a) + ar^3 = 0$	**M1**	**2.1**	Eliminate d	Could be a, a_1, g_1 or $1+\sqrt{5}$ but must now be consistent throughout (soi)
			$r^3 + 2r^2 - 1 = 0$	**A1**	**2.1**	Obtain correct cubic	
			f(– 1) = – 1 + 2 – 1 = 0 hence $(r + 1)$ is a factor	**B1**	**2.4**	Identify $(r + 1)$ as a factor, with justification	
			$(r + 1)(r^2 + r - 1) = 0$	**M1**	**3.1a**	Attempt to find all 3 roots of cubic	
			$r = -1, \frac{-1\pm\sqrt{5}}{2}$	**A1**	**1.1a**	For all three	
			GP is convergent so $-1 < r < 1$, so $r = \frac{-1+\sqrt{5}}{2}$	**B1**	**2.4**	Identify correct value of r, with reason	
			$S_\infty = \frac{1+\sqrt{5}}{1-\frac{1}{2}(-1+\sqrt{5})}$	**M1**	**1.1a**	Attempt sum to infinity, using their r	Need $-1 <$ their $r < 1$
			$= \frac{2(1+\sqrt{5})}{2-(-1+\sqrt{5})} = \frac{2(1+\sqrt{5})}{3-\sqrt{5}}$	**A1**	**2.1**	Simplify to correct expression	
			$= \frac{2(1+\sqrt{5})(3+\sqrt{5})}{(3-\sqrt{5})(3+\sqrt{5})} = \frac{2(3+\sqrt{5}+3\sqrt{5}+5)}{9-5}$	**M1**	**3.1a**	Rationalise denominator	Must also attempt expansion
			$= \frac{2(8+4\sqrt{5})}{4} = 4 + 2\sqrt{5}$ **AG**	**A1**	**2.1**	Obtain given answer www	
				[12]			

Question			Answer	Marks	AO	Guidance	
12	(i)		$\int(2\cos^2 4y-1)\mathrm{d}y=\int x^2\sin 2x\mathrm{d}x$	M1	1.1a	Separate variables	
			$2\cos^2 4y-1=\cos 8y$	M1	3.1a	Attempt use of double angle formula	Obtain $\pm\cos 8y$
			$\int\cos 8y\mathrm{d}y=\frac{1}{8}\sin 8y+c_1$	A1	1.1	Obtain correct integral	Condone no $+c_1$
			$\int x^2\sin 2x\mathrm{d}x=-\frac{1}{2}x^2\cos 2x+\int x\cos 2x\mathrm{d}x$	M1	3.1a	Attempt integration by parts once	
			$=-\frac{1}{2}x^2\cos 2x+\frac{1}{2}x\sin 2x-\int\frac{1}{2}\sin 2x\mathrm{d}x$	M1	1.1	Attempt second integration by parts	
			$=-\frac{1}{2}x^2\cos 2x+\frac{1}{2}x\sin 2x+\frac{1}{4}\cos 2x+c_2$	A1	1.1	Obtain correct integral	Condone no $+c_2$
			$\frac{1}{8}\sin 8y=-\frac{1}{2}x^2\cos 2x+\frac{1}{2}x\sin 2x+\frac{1}{4}\cos 2x+c$				
				[6]			
12	(ii)		$\sin\pi=\frac{8}{12}$ = $-4(\frac{1}{4})\cos\frac{1}{2}\pi+4(\frac{1}{4})\sin\frac{1}{2}\pi+2\cos\frac{1}{2}\pi+c$	M1	1.1	Attempt c, using $x=\frac{1}{4}\pi$, $y=\frac{1}{12}\pi$	
			$c=\frac{1}{2}\sqrt{3}-\pi$	A1	1.1	Obtain correct value for c for their correct equation Condone decimal equiv (– 2.276)	eg $c=\pi-\frac{1}{2}\sqrt{3}$ if their c on LHS, or $c=\pm\left(\frac{1}{16}\sqrt{3}\pi-\frac{1}{8}\right)$ if fractions not yet cleared
			$\sin 8y=\pi 2+\frac{1}{2}\sqrt{3}-$ $8y$ = (– 0.279), 3.421 y = (– 0.035), 0.428	M1	3.1a	Attempt positive value for y when x = 0	
			y = 0.428	A1 [4]	1.1	Obtain correct value for y	A0 if extra values