OCR
Oxford Cambridge and RSA

Practice Paper – Set 2

A Level Mathematics A

H240/02 Pure Mathematics and Statistics

MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 100

DRAFT

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Mark	AO	Guidance	
1	(i)		–4	B1 [1]	1.1		
1	(ii)		x, $(x + 2)$, $(x – 1)$	B1 [1]	1.1		
1	(iii)		$y = ax(x – 1)(x + 2)$ Subst (–1, –4) or from **(i)** $–4 = a(–1)(–2)(+1) \Rightarrow a = –2$	M1 M1 A1f [3]	3.1a 1.1 2.2a	 ft their **(i)** and **(ii)**	
1	(iv)		$y = –2x(x – 1)(x + 2)$ $y = –2x^3 – 2x^2 + 4x$ or $b = –2$, $c = 4$, $d = 0$	B1ft [1]	1.1	ft their **(ii)**	
2			Attempt $2^n – 1$ for any odd integer n eg $2^9 – 1 = 511$ This is a counter example as 511 is divisible by 7, hence claim false	M1 A1 M1 E1 [3]	3.1a 2.1 1.1 2.2a	 Any $2^{odd} – 1$ that is non-prime Counter example can be mentioned at the start	
3	(i)		Time = $\frac{500}{v}$, $T = \frac{500}{v} \times R$ Hence $T = \frac{500R}{v}$	 B1 [1]	 1.1	 **AG** Must see Time = $\frac{500}{v}$	
3	(ii)		$T = \frac{500}{v}\left(270 + \frac{v^3}{200}\right)$ $\left(= \frac{135000}{v} + \frac{5v^2}{2}\right)$ $\frac{dT}{dv} = -\frac{135000}{v^2} + 5v$ oe $-\frac{135000}{v^2} + 5v = 0$ $[v^3 = 27000]$ Required speed is 30 km/h	M1 M1 M1 A1 [4]	3.1a 3.4 1.1 3.2a	 Attempt diff their T Their $\frac{dT}{dv} = 0$ Allow $v = 30$ km/h; not just $v = 30$.	
3	(iii)		$T_{min} = \frac{135000}{30} + \frac{5 \times 30^2}{2}$ Min cost = £6750	M1 A1 [2]	1.1 3.2a	Subst their '30' into their T £ necessary	

Question			Answer	Mark	AO	Guidance	
4	(i)		Line $y = x$ drawn Any value in the range [0.70, 0.77]	M1 A1 [2]	1.1a 1.1	or implied by answer	
4	(ii)		eg cos 0.75= 0.731688869 Eg cos0.7390791171=0.7390891857 answer $x = 0.739$ (3 sf)	M1 M1 A1 [2]	1.1a 1.1 1.1	Any starting point. must be seen Any $x_{n+1} = \cos x_n$ where both x_{n+1} & x_n round to 0.7391, and anwer stated	x_1=0.75, x_2=0.7316…
5	(i)		Position vectors of midpoints AB & BC are $\begin{pmatrix}1.5\\0.5\\4\end{pmatrix}\begin{pmatrix}-1\\-0.5\\4\end{pmatrix}$ $2.5^2 + 1^2 (+ 0^2)$ Distance = $\frac{\sqrt{29}}{2}$	M1 A1 M1 A1 [4]	1.1a 1.1 1.1 1.1	Correct method for one midpoint Both midpoints correct ft their midpoints; √ not necessary for M1	
5	(ii)		$\overrightarrow{AB} = \begin{pmatrix}1\\-3\\2\end{pmatrix}$ $\overrightarrow{CD} = \begin{pmatrix}-4-x\\6\\3-z\end{pmatrix}$ $\overrightarrow{CD} = -2\overrightarrow{AB}$ $-x - 4 = -2 \Rightarrow x = -2$ $3 - z = -4 \Rightarrow z = 7$	M1 M1 A1 A1 [4]	3.1a 1.2 1.1 1.1	For scale factor –2	

6	**(i)**		**DR** $\tan\frac{\pi}{12} = \tan(\frac{\pi}{3} - \frac{\pi}{4})$ $= \frac{\sqrt{3}-1}{1+\sqrt{3}}$ oe $= \frac{\sqrt{3}-1}{1+\sqrt{3}} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$ $= \frac{4-2\sqrt{3}}{2}$ $= 2 - \sqrt{3}$ **(AG)**	**M1** **A1** **M1** **A1** **[4]**	**3.1a** **1.1a** **1.2** **2.1**	Any correct use of double angle formula Any correct expression for t (or correct QE) Attempts rationalising (or solve their QE) This form seen (or both roots) and correct answer alone	
6	**(ii)**		**DR** $\frac{\sqrt{3}}{2}\sin 3A - \frac{1}{2}\cos 3A = \frac{1}{4}$ $\sin(3A - 30°) = \frac{1}{4}$ $3A - 30° = 14.5$ $A = 14.8°$ or $3A - 30° = 165.5$ $A = 65.2$ (1 dp) or $3A - 30° = (14.5 + 360)°$ $A = 134.8^o$	**M1** **A1** **M1** **A1** **B1** **M1** **A1f** **[7]**	**1.1a** **3.1a** **1.1** **1.1** **2.4** **3.1a** **2.1**	Use of $\sin^{-1}$ both sides ft their 14.8° + 120°	

7			$V = 100h \Rightarrow \frac{dv}{dh} = 100$	**M1**	**3.4**		
			$\frac{dv}{dt} = \frac{dv}{dh} \times \frac{dh}{dt} = 100\frac{dh}{dt}$ $[= 25 - 4h^2]$	**A1**	**1.2**		
			$\Rightarrow 25 - 4h^2 = 100\frac{dh}{dt}$ oe	**M1**	**3.1b**	Equate $25 - 4h^2$ to their $\frac{dv}{dh} \times \frac{dh}{dt}$	
			$\Rightarrow \int_0^2 \frac{1}{25-4h^2}\,dh = \int_0^t \frac{1}{100}\,dt$	**M1**	**2.5**	Attempt integration with correct denominator on LHS	
			$\Rightarrow \frac{1}{10}\int_0^2 \frac{1}{5+2h} + \frac{1}{5-2h}\,dh = \int_0^t \frac{1}{100}\,dt$	**M1**	**3.4**	Attempt partial fractions with correct denominators on LHS	
				A1	**2.1**	Correct partial fractions	
			$\Rightarrow \frac{1}{10} \times \frac{1}{2}\left[\ln(5+2h) - \ln(5-2h)\right]_0^2 = \frac{t}{100}$	**M1**	**1.2**	Correct integral; ignore limits	
			$\Rightarrow 5\ln 9 = t$ oe	**A1**	**2.2a**	Any correct numerical expression for t	10.9861…
			Time when depth is 2 cm is 11.0 seconds (3 sf)	**A1**	**3.2a**	Allow 11 seconds	
				[9]			
8	**(i)**		$\mu = 56$	**B1**	**1.1a**		
			Percentage with masses > 59 g = 30%	**B1**	**1.1**	or 0.3	
				[2]			
8	**(ii)**		$\Phi\left(\frac{53-56}{\sigma}\right) = 0.3$, $\frac{53-56}{\sigma} = -0.5244$	**M1**	**2.1**		
			$\sigma = 5.721$	**A1**	**1.1**		
			$X \sim N(56, '5.721'^2)$ soi	**M1**	**2.4**	or $P(X > 65) = P(z > \frac{65-56}{5.721}) = P(z > 1.573)$	ft their σ
			$P(X > 65) = 0.0578$ or 5.78% (3 sf)	**A1**	**1.1**	Or **BC**	
				[4]			
8	**(iii)**		$P(X < 50) = 0.1471$	**M1**	**1.1a**		
			$P(X < a) = 0.0471$)	**A1**	**2.1**		
			$a = 46.4$ (3 sf)	**A1**	**1.1**		
				[3]			

9	**(i)**		$P(X = 3) = {}^5C_3 \times \frac{100}{300} \times \frac{99}{299} \times \frac{98}{298} \times \frac{200}{297} \times \frac{199}{296}$ $= 0.164318883 = 0.164$ (3 sf)	**M1** **A1** **[2]**	**1.1a** **1.1**	or equiv methods	
9	**(ii)**		P(disc is black) changes each trial (because no replacement) oe But change in prob is small oe Hence bin gives approx, but not exact, probs oe	**E1** **E1** **E1** **[3]**	**2.4** **2.4** **3.5b**		
9	**(iii)**		$P(X = 3$ using bin$) = {}^5C_3 \times (\frac{2}{3})^2(\frac{1}{3})^3$ $(= 0.164609053)$ $\frac{'0.164609053' - '0.164318883'}{'0.164318883'} \times 100$ $= 0.177\%$	 **M1** **A1** **[2]**	 **3.4** **1.1**	 ft their values for M1 cao	
9	**(iv)**		$\mu = 1000 \times 0.164609053$ $(= 164.609053)$ $\sigma^2 = 1000 \times 0.164609053 \times (1 - 0.164609053)$ $(= 137.5129127)$ $X \sim$ Normal $164.609053 \pm \sqrt{137.5129127}$ $= 152.88$ to 176.34 Estimated limits are 153 to 176	 **M1** **M1** **A1** **[3]**	 **3.3** **1.1a** **1.1**	 both *np* and *npq* correct method ft their μ and $\sqrt{\sigma^2}$. Allow rounding to 3 sf Allow (150 – 155) to (174 – 180)	 ft their 0.164.6.... Allow use of 0.1643.... $165 \pm \sqrt{138}$ or better

10	**(i)**		Method A: eg May not be representative, e.g. may have many (or few) appointments from one Dr Or may have many a.m. appts & few p.m. Method B: eg Time of 1st Dr's 1st & 2nd appointments may not be typical of his later ones. Similar for other Drs	**B1** **B1** **[2]**	**3.5b** **3.5b**	Or may not have a good spread of appts Or other sensible Or other sensible	
10	**(ii)**		Researcher suspects "more than" 10 mins	**B1** **[1]**	**2.4**	oe	
10	**(iii)**		0.99	**B1** **[1]**	**1.2**		
10	**(iv)**		H_0: $\mu = 10$ where μ is pop mean appointment time H_1: $\mu > 10$ $\bar{X} \sim N(10, \frac{3.4}{\sqrt{24}})$ and $X = \frac{285}{24}$ (= 11.875) $P(\bar{X} > 11.875) = 0.00345$ or better Compare with 0.01 Reject H_0 Sufficient evidence that mean time is > 10 mins	**B1** **B1** **M1** **A1** **M1** **M1** **A1** **[7]**	**1.1** **2.5** **3.3** **3.4** **1.1** **2.2b** **3.5a**	One error, eg undefined μ or 2-tail: **B0B1** May be implied or 0.003 **BC** In context. Not definite, eg "Mean time is > 10 mins": **A0**	
10	**(v)**		Times spent assumed normally distributed, hence sample mean also normally distributed	**B1** **[1]**	**1.2**	oe	

11	**(i)**		Points close to straight line with +ve gradient Hence 0.913 is the correct value	**B1** **B1** **[2]**	**2.4** **2.2b**	Dep on 1st B1	
11	**(ii)**		Sample is from one area, hence not random oe	**E2** **[2]**	**2.3** **2.2b**	Or might be different relationship elsewhere	Allow E1 for "Sample is small"
11	**(iii)**		Both depend on the size (of the pop) of area.	**E1** **[1]**	**2.4**		
11	**(iv)**		Correct point indicated (54200, 15300)	**B1** **[1]**	**3.2b**		
11	**(v)**		More local jobs (so higher proportion walk)	**E1** **[1]**	**2.2b**	Any sensible equivalent,	
11	**(vi)**	**(a)**	Eg Fewer walk to work	**E1** **[1]**	**2.2b**		
11	**(vi)**	**(b)**	Eg Some businesses within the borough have closed down or have moved to the outskirts	**E1** **[1]**	**2.2b**	or any relevant comment	

<table>
<tr>
<td>12</td><td>(i)</td><td>(a)</td>
<td>
<table>
<tr><td></td><td></td><td colspan="6">Biased die, Y</td></tr>
<tr><td></td><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td rowspan="6">Fair die, X</td><td>1</td><td>3</td><td>5</td><td>7</td><td>9</td><td>11</td><td>13</td></tr>
<tr><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td><td>14</td></tr>
<tr><td>3</td><td>5</td><td>7</td><td>9</td><td>11</td><td>13</td><td>15</td></tr>
<tr><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
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<tr><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr>
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<td>B2
[2]</td><td>1.1a
1.1</td><td>B1 for ≥ 30 values correct</td><td></td>
</tr>
<tr>
<td>12</td><td>(i)</td><td>(b)</td><td>Four 10s circled or otherwise indicated</td><td>B1
[1]</td><td>1.1</td><td>Must be exactly four 10s in table.</td><td></td>
</tr>
<tr>
<td>12</td><td>(ii)</td><td></td><td>Outcomes not equally likely oe</td><td>E1
[1]</td><td>2.3</td><td></td><td></td>
</tr>
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<td>12</td><td>(iii)</td><td></td><td>$\frac{1}{6}\times 0.14+\frac{1}{6}\times 0.14+\frac{1}{6}\times 0.1+\frac{1}{6}\times 0.01$
$= \frac{13}{200}$ or 0.065</td><td>M1
A1
[2]</td><td>3.1b
1.1</td><td></td><td></td>
</tr>
<tr>
<td>12</td><td>(iv)</td><td></td><td>P(S = 10 & one score = 4) =
$\frac{1}{6}\times 0.14+\frac{1}{6}\times 0.14+\frac{1}{6}\times 0.01$
P(One score = 4 | S = 10)
$= \frac{P(S=10 \text{ \& one score } =4)}{P(S=10)} = \frac{29}{600} \div \frac{13}{200}$
$= \frac{29}{39}$ or 0.744 (3 sf)</td><td>M1
M1
A1
[3]</td><td>3.1b
2.1
1.1</td><td>
ft their (iii), dep 1st M1 gained in (iv)
cao</td><td>($= \frac{29}{600}$ or 0.0483(3 sf))</td>
</tr>
<tr>
<td>12</td><td>(v)</td><td></td><td>n = 15</td><td>B1
[1]</td><td>1.2</td><td></td><td></td>
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