Oxford Cambridge and RSA

Practice Paper – Set 4

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 100

FINAL

This document consists of 16 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question includes the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1			$2^{3x-1}=3^{x+4} \Rightarrow 3x-1=\log_2(3^{x+4})$	**M1**	**1.1a**	Take logs of both sides – allow any (consistent) base including natural logs	
			$(3x-1)=(x+4)\log_2 3 \Rightarrow x=\ldots$	**M1**	**1.1**	Bring both powers to the front and attempt to make x the subject	
			$x=\dfrac{4\log_2 3+1}{3-\log_2 3}=5.19$	**A1**	**1.1**		In base 10 $x=\dfrac{4\log 3+\log 2}{3\log 2-\log 3}=5.19$
				[3]			

2			**DR**				
				M1	**1.1a**	Attempt to differentiate (all powers reduced by 1)	
			$\dfrac{dy}{dx}=2x^2+5x-3$	**A1**	**1.1**	Correct differentiation of all terms	
			$2x^2+5x-3>0 \Rightarrow (2x-1)(x+3)>0$	**M1**	**1.1**	Attempt to find critical values by any appropriate method (e.g. factorising, completing the square, quadratic formula)	
			$x<-3$ or $x>\frac{1}{2}$	**M1**	**1.1**	Choose 'outside region' for their critical values	
			$\{x:x<-3\}\cup\{x:x>\frac{1}{2}\}$	**A1**	**2.5**		
				[5]			

Question			Answer	Marks	AOs	Guidance	
3	(a)		$(x-a)^2+(y+a)^2=\ldots$ $\ldots=2a^2$	B1 B1 [2]	1.1 1.1	Correct LHS (accept if expanded: $x^2+y^2-2ax+2ay+2a^2$) Correct RHS Allow full marks for any equivalent form, e.g. $x^2+y^2-2ax+2ay=0$	
3	(b)		$(1-a)^2+(-5+a)^2=2a^2$ $a=\frac{13}{6}\Rightarrow \text{Area}=\pi\times2\left(\frac{13}{6}\right)^2$ $=\frac{169}{18}\pi$	M1 M1 A1 [3]	1.1a 1.1 2.2a	Substitute (1, –5) into their circle equation Solve for a and substitute into πr^2 with their r^2	
4			$(8p-3)-9p=5p-(8p-3)$ $p=3$ $a=27,\ d=-6$ $\frac{n}{2}\left[2(27)+(n-1)(-6)\right]=-1512$ $n^2-10n-504=0\Rightarrow(n-28)(n+18)=0$ $n=28$ only	M1 A1 A1FT M1 M1 A1 [6]	3.1a 1.1 1.1 2.1 1.1 2.2a	Setting up an equation to find p Using their value of p to calculate a and d Setting up an equation using the correct formula for the sum of an AP equated to –1512 Expand and attempt to solve 3-term quadratic equation in n This mark should be withheld if $n=-18$ appears as part of the final answer	Allow a single sign error Solving of 3-term quadratic may be done **BC**

Question			Answer	Marks	AOs	Guidance	
5	(a)		$\dfrac{1}{e^{2x}}=\dfrac{x}{x^2+3}\Rightarrow x^2+3=xe^{2x}$	M1	1.1	Equate expressions and cross-multiply (to remove fractions)	
			$x^2+3=xe^{2x}\Rightarrow x^2+3-xe^{2x}=0$	A1	2.2a	**AG** – sufficient working must be shown to indicate that result has been derived correctly	
				[2]			
5	(b)		$\dfrac{d}{dx}(xe^{2x})=e^{2x}+2xe^{2x}$	M1	1.1	Attempt at product rule for xe^{2x} – expression must be of the form $\pm e^{2x}(1\pm kxe^{2x})$	
			$h'(x)=2x-e^{2x}-2xe^{2x}$	A1	1.1		
			$x_{n+1}=x_n-\dfrac{x_n^2+3-x_ne^{2x_n}}{2x_n-e^{2x_n}-2x_ne^{2x_n}}$	M1*	2.1	Correct application of NR with their $h'(x)$	
			$x_{n+1}=\dfrac{2x_n^2-x_ne^{2x_n}-2x_n^2e^{2x_n}-x_n^2-3+x_ne^{2x_n}}{2x_n-e^{2x_n}(1+2x_n)}$	M1dep*	1.1	Correctly combining as a single fraction and expanding any brackets in numerator	
			$x_{n+1}=\dfrac{x_n^2-2x_n^2e^{2x_n}-3}{2x_n-e^{2x_n}(1+2x_n)}=\dfrac{x_n^2\left(1-2e^{2x_n}\right)-3}{2x_n-(1+2x_n)e^{2x_n}}$	A1	2.2a	**AG –** sufficient working must be shown as the answer is given	
				[5]			
5	(c)		From graph eg $x_1=1$	M1	3.1a	Suitable starting value chosen	
			$x_2=0.83195181...,\quad x_3=0.7754682...$	A1	1.1	At least two correct applications of NR	
			$x_4=0.77016...,\quad x_5=0.77011...$ $\alpha=0.770$ (correct to 3 dp)	A1	1.1		
				[3]			

Question			Answer	Marks	AOs	Guidance	
5	(d)		**DR**				
			$\text{fg}(x) = \text{f}\left(\text{e}^{-2x}\right) = \dfrac{\text{e}^{-2x}}{\left(\text{e}^{-2x}\right)^2 + 3}$	**M1**	**2.1**	Attempt at fg(x) – need not be simplified	
			$2\text{e}^{-4x} - 13\text{e}^{-2x} + 6 = 0$	**A1**	**1.1**	Correct equation – fractions must be removed and powers simplified	Or equivalent
			$k = \text{e}^{-2x}$	**M1***	**3.1a**	Substitute for e^{-2x} (or equivalent)	**Alternatively:** Factorise into two brackets containing e^{-2x} **M2**
			$(2k-1)(k-6) = 0$	**M1dep***	**1.1**	Attempt to solve resulting quadratic	
			$\text{e}^{-2x} = \frac{1}{2} \Rightarrow x = -\frac{1}{2}\ln\left(\frac{1}{2}\right)$	**A1**	**2.2a**	**www oe**	
			$\text{e}^{-2x} \neq 6$ Q it is given that $x \geq 0$	**A1**	**2.3**	Correct statement or equivalent that e^{-2x} cannot be greater than 1	
				[6]			

Question			Answer	Marks	AOs	Guidance	
6	(a)		$x=\ln(t^2-4)\Rightarrow\dfrac{dx}{dt}=\dfrac{2t}{t^2-4}$	M1	1.1	Attempt differentiation of x using chain rule – must be of the form $\dfrac{kt}{t^2-4}$	
			Area $=\displaystyle\int\frac{4}{t^2}\left(\frac{2t}{t^2-4}\right)dt$	M1	1.1a	Use of $\displaystyle\int y\frac{dx}{dt}\,dt$ with their $\dfrac{dx}{dt}$	
			$=\displaystyle\int\frac{8}{t(t^2-4)}\,dt$	A1	2.2a	**AG**	
			$a=3,\ b=4$	B1	2.2a	Correct limits	
				[4]			
6	(b)		**DR**				
			$\dfrac{8}{t(t^2-4)}\equiv\dfrac{A}{t}+\dfrac{B}{t-2}+\dfrac{C}{t+2}$	B1	3.1a	Correct form of partial fractions	
			$8=A(t-2)(t+2)+Bt(t+2)+Ct(t-2)$	M1	1.1a	Cover up, substituting or equating coefficients – must be a complete method for finding one of A, B or C	
			$A=-2,\ B=1,\ C=1$	A2	1.1, 1.1	**A1** for one correct	
			$\displaystyle\int\left(-\frac{2}{t}+\frac{1}{t-2}+\frac{1}{t+2}\right)dt=-2\ln t+\ln(t-2)+\ln(t+2)$	M1*	1.1	Attempt to integrate all three terms – must be of the form $\alpha\ln t+\beta\ln(t-2)+\gamma\ln(t+2)$	
			$(-2\ln 4+\ln 2+\ln 6)-(-2\ln 3+\ln 1+\ln 5)$	M1dep*	1.1	Applying their limits correctly	
				M1	2.1	Correctly combining all their log terms – dependent on both previous M marks	
			$\ln\left(\frac{27}{20}\right)$	A1	2.2a	$k=\frac{27}{20}$	
				[8]			

Question			Answer	Marks	AOs	Guidance	
6	(c)		$t^2 = \frac{4}{y} \Rightarrow x = \ln\left(\frac{4}{y} - 4\right)$	M1*	3.1a	Re-arrange and eliminate t	
			$e^x = \frac{4}{y} - 4 \Rightarrow y = \mathrm{K}$	M1dep*	1.1	Remove logs and attempt to make y the subject	
			$y = \frac{4}{e^x + 4}$	A1	1.1		
			Alternative solution				
			$e^x = t^2 - 4$	M1*		Remove logs	
			$t^2 = e^x + 4 \Rightarrow y = \mathrm{K}$	M1dep*		Rearrange and eliminate t	
			$y = \frac{4}{e^x + 4}$	A1			
				[3]			

Question			Answer	Marks	AOs	Guidance	
7	(a)		$\mathbf{s} = 4(2\mathbf{i}+4\mathbf{j}) - \frac{1}{2}(4)^2(3\mathbf{i}-5\mathbf{j})$	M1	3.3	Attempt use of $\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$	Accept equivalent full methods using *suvat* equations e.g. first using $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ to find $\mathbf{u}$ and then using $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
			$\mathbf{s} = \left(-16\mathbf{i} + 56\mathbf{j}\right)\text{m}$	A1	1.1		
				[2]			
7	(b)		$2\mathbf{i} + 4\mathbf{j} = \mathbf{u} + 4(3\mathbf{i} - 5\mathbf{j})$	M1*	3.3	Attempt use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$	
			$\mathbf{u} = -10\mathbf{i} + 24\mathbf{j}$	A1	1.1		
			$\lvert \mathbf{u} \rvert = \sqrt{(-10)^2 + 24^2}$	M1dep*	1.1	Attempt magnitude of their $\mathbf{u}$	
			$= 26\,\text{m}\,\text{s}^{-1}$	A1	2.2a		
				[4]			

8	(a)			M1	3.1b	Attempt moments e.g. about A	
			$2(150\cos\theta) + x(750\cos\theta) = 4(R_B \sin\theta)$	A2	1.1, 1.1	**A1** for any two terms correct	R_B is the normal contact force at the wall
			$R_B = F_A \Rightarrow F_A = \frac{25}{2}(2+5x)$	A1	1.1	**AG** – sufficient working must be shown to justify the given answer	F_A is the frictional force at the ground
				[4]			
8	(b)		$R_A = 150 + 750$	B1	3.3	Resolving vertically	R_A is the normal contact force at the ground
			$\frac{25}{2}(2+5x) \le \frac{1}{4}(900)$	M1	3.4	Correct use of $F_A \le \mu R_A$	Allow equals throughout
			$x \le 3.2$ so the greatest value of x is 3.2	A1	2.2a	Must have maximum value of x explicitly stated.	
				[3]			

Question			Answer	Marks	AOs	Guidance	
9	(a)		$\frac{\mathrm{d}v}{\mathrm{d}t} = 8t^3 + 2kt$	**B1**	**1.1**	Correct expression for the acceleration	
			$8(2)^3 + 2k(2) = 28$	**M1**	**1.1**	Substitute t = 2 into their a and equate to 28	
			$4k = 28 - 64 \Rightarrow k = -9$	**A1** **[3]**	**2.2a**	**AG**	
9	(b)		$\frac{\mathrm{d}v}{\mathrm{d}t} = 0 \Rightarrow 2t(4t^2 - 9) = 0$	**M1**	**3.1b**	Substituting the correct value of k and equating to zero	
			$t = 1.5$ (and $t = 0$)	**A1**	**1.1**	**AG** Correctly finding the given value of t	
			E.g. $\left.\frac{\mathrm{d}^2v}{\mathrm{d}t^2}\right\vert_{t=1.5} = 24(1.5)^2 - 18 > 0$ so a minimum	**B1** **[3]**	**2.1**	Showing that this value of t gives a minimum	Or complete argument from the shape of the curve, or from first derivatives
9	(c)		$s = \frac{2}{5}t^5 - 3t^3 - 4t$ $(+c)$	**M1***	**1.1a**	Attempt to integrate v (all powers increased by 1)	Constant not required for this first M mark
			$-6.4125 = 0.4 - 3 - 4 + c \Rightarrow c = \ldots$	**M1dep***	**2.1**	Attempt to find c	$c = 0.1875$
			$s = 0.4(1.5)^5 - 3(1.5)^3 - 4(1.5) + 0.1875$	**M1**	**1.1**	Substitute 1.5 into their expression for s – dependent on both previous M marks	
			$s = -12.9$ so distance of P from O is 12.9 m	**A1** **[4]**	**3.2a**		

Question			Answer	Marks	AOs	Guidance	
10	(a)		Acceleration component $= g\sin 30°$	B1	1.2	Correct acceleration component seen	
			$v_M^2 = 4.2^2 + 2(g\sin 30°)x$	M1	3.3	Use of $v^2 = u^2 + 2as$ for the motion from A to M	x is the distance AM and v_M is the speed of P at M
			$R = mg\cos 30°$	B1	3.3	Resolving perpendicular to the plane	R is the normal contact force between P and the plane, m is the mass of P
			$F = \frac{\sqrt{3}}{6}mg\cos 30°$	M1	3.4	Use of $F = \mu R$ for the motion of P between M and B	
			$mg\sin 30° - F = ma$	M1*	3.3	Use of Newton's 2nd Law for the motion of P between M and B	
			$12.6^2 = v_M^2 + 2g\left(\sin 30° - \frac{\sqrt{3}}{6}\cos 30°\right)(20 - x)$	M1dep*	3.4	Correct use of $v^2 = u^2 + 2as$ for the motion from M to B with their a and correct s	
			$12.6^2 = 4.2^2 + 2(g\sin 30°)x$ $+ 2g(20-x)\left(\sin 30° - \frac{\sqrt{3}}{6}\cos 30°\right)$	M1	2.1	Substitute their expression for v_M to obtain an equation in x only	
			$x = 8.8$ so the distance AM is 8.8 m	A1 [8]	2.2a	**BC**	
10	(b)		$\tan\alpha = \frac{R}{F} = \frac{mg\cos 30°}{\frac{\sqrt{3}}{6}mg\cos 30°}$	M1*	3.1b	Equates ratio of contact forces to tan	
			angle = $180° - \alpha$	M1dep*	1.1		
			$= 106.1°$	A1 [3]	1.1	Correct answer (to at least 3 sf)	106.102 113…

Question			Answer	Marks	AOs	Guidance	
11	(a)	(i)	$0=(V\sin\alpha)^2+2(-g)H$	M1	3.3	Use of $v^2=u^2+2as$ vertically	
			$H=\frac{V^2}{2g}\sin^2\alpha$	A1	1.1	**AG** – sufficient working must be shown	
				[2]			
11	(a)	(ii)	$R=(V\cos\alpha)t$ and $0=(V\sin\alpha)t+\frac{1}{2}(-g)t^2$	M1*	3.3	Use of $s=ut+\frac{1}{2}at^2$ horizontally and vertically	**Alternatively:** Finding t from $0=V\sin\alpha-gt$ and using double the value, oe
			$(V\sin\alpha)-\frac{1}{2}g\left(\frac{R}{V\cos\alpha}\right)=0$	M1dep*	1.1	Re-arranging and eliminating t	
			$gR=2V^2\sin\alpha\cos\alpha\Rightarrow R=\frac{V^2}{g}\sin 2\alpha$	A1	2.2a	**AG** – sufficient working must be shown	
				[3]			
11	(b)		$R_0=\frac{V^2}{g}$	B1	3.3	Correct expression for maximum range	
			$\sin^2\alpha=\frac{2gH}{V^2},\ \cos^2\alpha=\frac{V^2-2gH}{V^2}$	M1*	3.1a	Obtain expressions for $\sin^2\alpha$ and $\cos^2\alpha$ using $\sin^2\theta+\cos^2\theta\equiv1$	
			$R=\frac{2V^2}{g}\times\frac{\sqrt{2gH}}{V}\times\frac{\sqrt{V^2-2gH}}{V}$	M1dep*	2.1	Eliminate α	
			$R=\frac{2}{g}\sqrt{2gH}\sqrt{R_0g-2gH}$				
			$\Rightarrow R^2=\frac{4}{g^2}(2gH)(R_0g-2gH)$	M1	3.4	Eliminate V using maximum range and remove square roots – dependent on both previous M marks	
			$\Rightarrow 16H^2-8R_0H+R^2=0$	A1	2.2a	**AG** – sufficient working must be shown	
				[5]			

Question			Answer	Marks	AOs	Guidance	
11	(c)	(i)	$16H^2 - 1600H + 36\,864 = 0$	M1	1.1	Substitute given values to obtain a quadratic in H	
			$H = 64\text{ m}$ or 36 m	A1	1.1	**BC**	
				[2]			
11	(c)	(ii)	$\sin 2\alpha = \frac{192}{200}$	M1	1.1	Use given values to obtain a trigonometric equation in α	
			$\alpha = 36.9°$	A1	1.1	0.644 rad	
			$\alpha = 53.1°$	A1	1.1	0.927 rad	
				[3]			
11	(d)		The model has not considered the possibility of: • Air resistance • The ball would have dimensions • Wind • The possible spin of the ball	B1 [1]	3.5b	Any one correct limitation identified	

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