Oxford Cambridge and RSA

Practice Paper – Set 2

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 100

Version **Pre SSU**
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This document consists of 16 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question includes the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

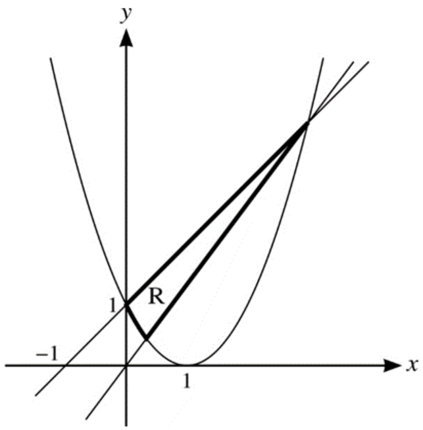
f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1				B1 B1 B1 B1 [4]	1.1 1.1 1.1 1.1	$y=(x-1)^2$ drawn correctly $3y=4x$ drawn correctly $y-x=1$ drawn correctly Correct identification of region (dependent on previous B marks); condone identification via shading so long as there is no ambiguity about the intended region	x-axis must be a tangent to the curve Line must pass through the origin Positive gradient and y-intercept Note that both lines and curve must meet at the same point for this final mark to be awarded (ignore labelling on axes)

Question			Answer	Marks	AOs	Guidance	
2	(i)		$r = \frac{3}{4}$	B1	1.1a		
			$u_5 = 12r^4$	M1	1.2	Applying their r in the correct formula for u_5 with $a = 12$	Or repeated use of their r
			$u_5 = \frac{243}{64}$	A1	1.1		3.796 875
				[3]			
2	(ii)		$S_\infty = \dfrac{12}{1-\frac{3}{4}}$ or $S_N = \dfrac{12\left(1-(\frac{3}{4})^N\right)}{1-\frac{3}{4}}$	B1ft	1.1	Correctly applying formula for S_∞ or S_N with their value of r	
			$\dfrac{12}{1-\frac{3}{4}} - \dfrac{12\left(1-(\frac{3}{4})^N\right)}{1-\frac{3}{4}} \le 0.0096$	M1	2.1	Attempt at $S_\infty - S_N$ compared with 0.0096 (dependent on previous B1)	Accept any inequality or equals for this mark
			$48 - 48\left(1-(\frac{3}{4})^N\right) \le 0.0096 \Rightarrow \left(\frac{3}{4}\right)^N \le 0.0002$	A1	2.2a	**AG** — completely correct working	
				[3]			
2	(iii)		$N \log\left(\frac{3}{4}\right) \le \log(0.0002) \Rightarrow N \ge \ldots$	M1	1.1	Take logs and attempt to make N the subject (accept any inequality or equals for this mark)	$N = \log_{\frac{3}{4}}(0.0002)$
			$N \ge 29.6\ldots \Rightarrow N = 30$	A1	2.2a		
				[2]			

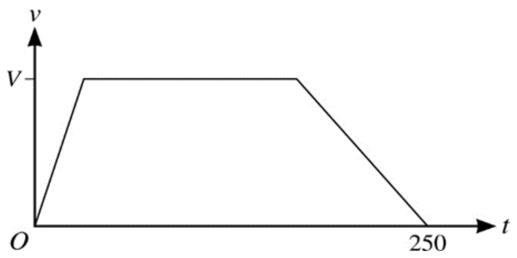
Question			Answer	Marks	AOs	Guidance	
3			Reflection, stretch and translation	**B1**	**2.5**	All three correct	Do not accept any other wording
			(reflection) in the line $y = x$	**B1**	**1.1**		
			(stretch) scale factor $\frac{1}{3}$ parallel to the x-axis	**B1**	**1.1**	Accept 'in the x-direction'; accept 'factor' or 'SF' for 'scale factor'	Do not accept 'in/on/across/up the x-axis' or '$\frac{1}{3}$ units'
			(translation) $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$	**B1**	**1.1**	Accept '5 units in the negative y-direction' or '–5 units parallel to the y-axis' Order of transformations must be correct for all 4 marks to be awarded	Do not accept 'in/on/across/up the y-axis'
				[4]			

Question			Answer	Marks	AOs	Guidance	
4	(i)		$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$, where $\frac{dy}{dt} = 3t^2$, $\frac{dx}{dt} = 2t$	*M1	1.1a	Correct application of $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ with their $\frac{dy}{dt}$ and $\frac{dx}{dt}$ (with at least one correct)	
			$\frac{dy}{dx} = \frac{3t^2}{2t} \quad \left(= \frac{3}{2}t\right)$	A1	1.1	Correct derivative (need not be simplified at this stage)	
			$y - t^3 = \frac{3t^2}{2t}(x - t^2)$	dep*M1	1.1	Use of $y - t^3 = m(x - t^2)$ with their m in terms of t	
			$2y = 3tx - t^3$	A1	2.2a	**AG**	
				[4]			
4	(ii)		**DR**				
			Substitute A giving $t^3 - 3t\left(\frac{19}{12}\right) + 2\left(-\frac{15}{8}\right) = 0$, and attempt factor theorem with $f(t) = 4t^3 - 19t - 15$, oe	M1	3.1a	Correctly substitute A into given tangent and attempt to find a factor	
			$f(-1) = 0 \Rightarrow (t+1)$ is a factor	A1	1.1		
			$f(t) = (t+1)(4t^2 - 4t - 15)$	M1	1.1	Attempt to obtain a quadratic factor	By any correct method
			$f(t) = (t+1)(2t-5)(2t+3)$	A1	1.1		
			$t = \frac{5}{2}$ only, as $t \geq 0$	A1	3.2a		
			$y = 0 \Rightarrow B\left(\frac{25}{12}, 0\right)$ and area $= \frac{1}{2} \times \frac{15}{8} \times \frac{25}{12}$	M1	1.1	Use of their t to find B and attempt to find area using their B	Their value of t must be positive
			area $= \frac{125}{64}$	A1	2.2a		
				[7]			

Question			Answer	Marks	AOs	Guidance	
5	(i)		**DR**				
			Attempt product rule for y	**M1**	3.1a	Attempt must be of the form $(ax+b)e^{-x} \pm (cx^2+dx)e^{-x}$	
			$y' = (4x-3)e^{-x} - (2x^2-3x)e^{-x}$	**A1**	1.1	Correct derivative, in any form	
			$y' = 0 \Rightarrow (4x-3)-(2x^2-3x) = 0$	**M1**	2.1	Set $y'=0$ and eliminate exponentials	
			Obtain quadratic in x and attempt to solve	**M1**	1.1	Dependent on both previous M marks	$2x^2-7x+3=0$
			$x=\frac{1}{2}$, $x=3$	**A1**	1.1	Correct values from correct equation	
			$-e^{-\frac{1}{2}} \le y \le 9e^{-3}$	**A1**	2.5	Correct range, including correct inequality signs and either y, f or f(x) used for range notation (not x)	Allow 'closed interval' notation $[-e^{-\frac{1}{2}}, 9e^{-3}]$
				[6]			
5	(ii)		**DR** $k=3$	**B1ft**	2.3	FT their larger value of x from **(i)**	
				[1]			
5	(iii)		Use integration by parts with $u=2x^2-3x$ and $v'=e^{-x}$	**M1**	1.1	Must obtain result $f(x) \pm \int g(x)\,dx$	
			$\int (2x^2-3x)e^{-x}\,dx = -(2x^2-3x)e^{-x} + \int (4x-3)e^{-x}\,dx$	**A1**	1.1		
			Attempt parts again with $u=ax+b$ and $v'=e^{-x}$	**M1**	1.1	Dependent on previous M mark	
			$\int (2x^2-3x)e^{-x}\,dx = -(2x^2+x+1)e^{-x}$ $(+c)$	**A1**	1.1	oe; accept unsimplified (but all bracketing must be correct)	
			$2x^2-3x=0 \Rightarrow x=\frac{3}{2}$ (and $x=0$)	**B1**	3.1a		
			Correct use of correct limits	**M1**	1.1	Dependent on both previous M marks	
			Integral is $1-7e^{-\frac{3}{2}} < 0$ so area is $7e^{-\frac{3}{2}}-1$	**A1**	2.2a		
				[7]			

Question			Answer	Marks	AOs	Guidance	
6	(i)		$p=3$	B1	1.1		
			$q=1$	B1	1.1		
				[2]			
6	(ii)		$x-3=\tan u \Rightarrow \mathrm{d}x=\sec^2 u\ \mathrm{d}u$	M1	1.1	Attempt to connect dx and du	
			$\int_3^4 \frac{1}{x^2-6x+10}\,\mathrm{d}x=\int_0^{\frac{1}{4}\pi}\frac{\sec^2 u}{\tan^2 u+1}\,\mathrm{d}u$	A1ft	1.1	Correct integral and limits; ft their p	
			$=\int_0^{\frac{1}{4}\pi}\mathrm{d}u=\frac{1}{4}\pi$	A1	1.1		
				[3]			
6	(iii)		Integral is $\frac{1}{2}\int_3^4\frac{2x-6}{x^2-6x+10}\,\mathrm{d}x+\int_3^4\frac{3}{x^2-6x+10}\,\mathrm{d}x$	B1	2.1	Or with single numerator $2x-6+6$	Limits not required
			Use of $\int\frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x$ and their answer to part **(ii)**	*M1	3.1a		
			$\frac{1}{2}\left[\ln(x^2-6x+10)\right]_3^4+\frac{3}{4}\pi$	A1ft	1.1	FT 3 × their answer to part **(ii)**	
			Correct use of limits and correct use of logs	dep*M1	1.1		
			$\frac{3}{4}\pi+\ln\sqrt{2}$	A1	2.2a		
			Alternative solution				
			$\int_0^{\frac{1}{4}\pi}\frac{3+\tan u}{\tan^2 u+1}\sec^2 u\,\mathrm{d}u$	*M1		Attempt use of previous substitution	Limits not required
			$\int_0^{\frac{1}{4}\pi}(3+\tan u)\,\mathrm{d}u$	A1		Correct integral and limits	
			$=\left[3u-\ln(\cos u)\right]_0^{\frac{1}{4}\pi}$	A1			
			Correct use of limits and correct use of logs	dep*M1			
			$\frac{3}{4}\pi+\ln\sqrt{2}$	A1			
				[5]			

Question			Answer	Marks	AOs	Guidance	
7			$\begin{pmatrix} 3 \\ -2a \end{pmatrix} + \begin{pmatrix} 2b \\ 3a \end{pmatrix} + \begin{pmatrix} -2 \\ b \end{pmatrix} = \mathbf{0}$	**M1**	**3.3**	Equilibrium $\Rightarrow \sum \mathbf{F}_i = \mathbf{0}$	Either correct equation in *a* and/or *b* can imply this mark
			$3 + 2b - 2 = 0 \Rightarrow b = -0.5$	**A1**	**1.1**		
			$-2a + 3a + b = 0 \Rightarrow a = 0.5$	**A1**	**1.1**		
				[3]			

Question			Answer	Marks	AOs	Guidance	
8	(i)			B1 B1 [2]	3.1b 3.1b	Correct trapezium shape Axes labelled correctly with V and 250 marked	Must start at the origin and stop on the t axis
8	(ii)		Acceleration 0.4 and deceleration 0.08 give time intervals of 2.5V and 12.5V respectively	B1	3.4	Either of these intervals obtained	
			Three time intervals: 2.5V, 250 – 15V and 12.5V	M1	3.4	Three time intervals found, in terms of V, summing to 250	At least one interval must be correct
			Area under their v-t graph = 880	M1	1.1	Attempt at area of trapezium or two triangles plus rectangle	
			$\frac{1}{2}\times 2.5V \times V + V(250-15V)+\frac{1}{2}\times 12.5V \times V = 880$	A1	2.1	oe; unsimplified correct equation in V	
			Attempt simplification to required form	M1	1.1	Dependent on both previous M marks; must correctly remove fractions in their equation and obtain result of the form $aV^2+bV+c=0$	
			$3V^2-100V+352=0$	A1 [6]	2.2a	**AG**	
8	(iii)		$(3V-88)(V-4)=0$	M1	1.1	Attempt to solve their 3-term quadratic	Or **BC**
			$V=\frac{88}{3}$ or $V=4$	A1	1.1		
			$V=4$ only, as $V=\frac{88}{3}$ is far too high (e.g. it equates to covering a distance of 100 m in less than 4 seconds)	A1 [3]	3.2b	Or 'mathematical' reason (e.g. time taken to decelerate from larger speed is greater than 250 s, so not possible)	

Question			Answer	Marks	AOs	Guidance	
9	(i)		$\dot{\mathbf{r}}_A = 2t\mathbf{i} + 3\mathbf{j}$	B1	1.1		
			$\dot{\mathbf{r}}_B = -4t\mathbf{i} + (3-4t)\mathbf{j}$	B1	1.1		
			$(2t)^2 + 9 = (-4t)^2 + (3-4t)^2$	M1	3.1a	$\lvert\dot{\mathbf{r}}_A\rvert = \lvert\dot{\mathbf{r}}_B\rvert$ with/without square root	
			$7t^2 - 6t = 0 \Rightarrow t = \ldots$	M1	1.1	Expand and attempt to solve quadratic in t (to obtain two solutions)	
			$t = 0$ or $t = \frac{6}{7}$	A1	1.1	Both values of t must be given	
				[5]			
9	(ii)		$\mathbf{r}_A - \mathbf{r}_B = (3t^2 - 1)\mathbf{i} + (-1 + 2t^2)\mathbf{j}$	*M1	3.1a	Consider $\pm(\mathbf{r}_A - \mathbf{r}_B)$	Condone one sign error
			$d^2 = (3t^2 - 1)^2 + (-1 + 2t^2)^2$	dep*M1	1.1	Use of $d^2 = \lvert\mathbf{r}_A - \mathbf{r}_B\rvert^2$	
			$= 9t^4 - 6t^2 + 1 + 4t^4 - 4t^2 + 1 = 13t^4 - 10t^2 + 2$	A1	2.2a	**AG** Expand correctly to given answer	Must show at least one intermediate step
				[3]			
9	(iii)		$\frac{\mathrm{d}}{\mathrm{d}t}(d^2) = 52t^3 - 20t$	B1	3.1a		
			$\frac{\mathrm{d}}{\mathrm{d}t}(d^2) = 0 \Rightarrow t = \ldots$	*M1	2.1	Set their derivative = 0 and solve for t	
			$t = 0$ and $t = \sqrt{\frac{5}{13}}$	A1	1.1	Both values correct; accept 0.620…	Ignore any mention of negative values of t
			Test nature of stationary point with correct value(s) of t	B1	2.1	e.g. $\frac{\mathrm{d}^2}{\mathrm{d}t^2}(d^2) = 156t^2 - 20 > 0$ when $t^2 = \frac{5}{13}$ so minimum	Or any other valid method
			Substsitute their non-zero t into d or d^2	dep*M1	1.1		
			$d = \frac{1}{\sqrt{13}}$ or 0.277	A1	2.2a	Dependent on all previous marks	0.277 350…
				[6]			

Question			Answer	Marks	AOs	Guidance	
10	(i)		Moments about A: $Wa\cos 30° + 2W(2a\cos 30°) = 2R_B a$	M1	3.3	Correct number of terms and attempt at component/perp. dist. for W and $2W$	
			$R_B = \frac{5}{4}\sqrt{3}\,W$	A1	1.1		
			Resolve vertically: $R_A + R_B\cos 30° = W + 2W$	M1	3.3	Four terms, with attempt at component of the force at B	
			$R_A = \frac{9}{8}W$	A1ft	1.1	Consistent with their R_B	
			Resolve horizontally: $F_A = R_B\sin 30°$	B1	3.3		$F_A = \frac{5}{8}\sqrt{3}\,W$
			$F_A \le \mu R_A \Rightarrow \mu \ge \ldots$	M1	3.3	Dependent on all previous M marks	Allow equals here
			$\mu \ge \frac{5}{9}\sqrt{3}$ so the least value of μ is $\frac{5}{9}\sqrt{3}$	A1	2.2a		
			Alternative solution				
			Moments about B: $Wa\cos 30° + F_A(2a\sin 30°) = R_A(2a\cos 30°)$	M1		Correct number of terms and attempt at components/perp. distances	
			Resolve along AB: $R_A\cos 60° + F_A\cos 30° = W\cos 60° + 2W\cos 60°$	M1		Four terms, with components attempted	
			Both equations correct	A1		Unsimplified	
			Solve simultaneous equations for R_A and F_A	M1		Dependent on both previous M marks	
			$R_A = \frac{9}{8}W$ and $F_A = \frac{5}{8}\sqrt{3}\,W$	A1		Both correct, from correct equations	
			$F_A \le \mu R_A \Rightarrow \mu \ge \ldots$	M1		Dependent on all previous M marks	Allow equals here
			$\mu \ge \frac{5}{9}\sqrt{3}$ so the least value of μ is $\frac{5}{9}\sqrt{3}$	A1			
				[7]			
10	(ii)		$R_A^2 + F_A^2 = 39$	M1	3.4	Use of their R_A and F_A in terms of W in equation for magnitude of resultant	
			$W = 4$	A1 [2]	1.1	cao	

Question			Answer	Marks	AOs	Guidance	
11	(i)		**DR**				
			$(25\sin\alpha)t - 4.9t^2 = (15\sin 2\alpha)t - 4.9t^2$	**M1**	**3.3**	Use $s = ut + \frac{1}{2}at^2$ for both, and equate	
			$25\sin\alpha = 15\sin 2\alpha$	**A1**	**1.1**		
			$25\sin\alpha = 30\sin\alpha\cos\alpha \Rightarrow \cos\alpha = \ldots$	**M1**	**2.1**	Correct use of double angle formula and attempt to solve for $\cos\alpha$	
			$\cos\alpha = \frac{5}{6}$ $\left(\text{and } \sin\alpha = \frac{1}{6}\sqrt{11}\right)$	**A1**	**1.1**		$\alpha = 33.557\ldots°$
			$(25\cos\alpha)t + (15\cos 2\alpha)t = 72 \Rightarrow t = \ldots$	**M1**	**3.3**	Use $s = ut$ for both, equate total to 72 and attempt to solve for t	
			$t = 2.7$	**A1**	**2.2a**		
			Height of C is $(25\sin\alpha)t - 4.9t^2 = 1.59$ m	**A1**	**3.4**		1.591 028 8…
				[7]			
11	(ii)		**DR** $v_h = 25\cos\alpha$	**B1ft**	**3.4**	With their value of $\cos\alpha$	$v_h = 20.833\ldots$
			$v_v = 25\sin\alpha - 9.8t$	**B1ft**	**3.4**	With their values of $\sin\alpha$ and t	$v_v = \pm 12.640\ldots$
			$\tan\theta = \frac{v_v}{v_h}$	**M1**	**3.1a**	θ is angle with horizontal; condone sign error/ambiguity for this mark	
			Direction is 31.2° below the horizontal	**A1**	**3.2a**		31.247 93…
				[4]			
11	(iii)		e.g. include the dimensions of the footballs in the model of the motion e.g. use a more accurate value of g in the model of the motion e.g. include air resistance in the model of the motion	**B1** **[1]**	**3.5c**	**DR**	

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