Ladder and supported beam problems EXTRA 1 (QNS & ANS)

Ladder problems

1. A ladder of length 4 m and weight W N rests in equilibrium with its foot A on horizontal ground and resting against a vertical wall at the top B. The leader is uniform; contact with the wall is smooth but contact with the ground is rough and the coefficient of friction is $\frac{1}{3}$. Find the angle θ between the ladder and the wall when the ladder is on the point of slipping.

2. A ladder of length $2a$ and weight W rests with its foot, A, on rough horizontal ground where he coefficient of friction is $\frac{3}{4}$. The top of the ladder, B, rests against a vertical wall and a painter, of weight $2W$, is standing at the top of the ladder. By modelling the ladder as a uniform rod, the wall as smooth and the painter as a particle, find the angle θ between the ladder and the wall when the ladder is just about to slip.

3. A uniform ladder of weight 200 N and length 2 m rests with one end, A, on level ground and the other end B resting against a vertical wall. When the ladder is in limiting equilibrium, the angle between the ladder and the wall is θ.
   (a) If contact between the wall is smooth, contact with the ground is rough and he coefficient of friction is $\frac{1}{3}$, find θ to the nearest degree.
   (b) If the ground is rough, the wall is smooth and $\theta = 45°$, find:
      (i) the normal reaction with the ground
      (ii) the frictional force,
      (iii) the value of the coefficient of friction.
   (c) If contact is rough with both wall and ground, and the coefficient of friction is $\frac{1}{4}$ in both case, find θ. (Remember that the ladder will not slip until friction is limiting in both cases).
   (d) Contact with the wall is smooth, and contact with the ground is rough. When $\theta = 60°$ a workman of weight 80 kg can climb one quarter of the way up the ladder before limiting equilibrium is reached. Find the reaction at the wall and the coefficient of friction. Take g as 10 and treat the workman as a point load.
   (e) If, in question (d), θ is reduced to $30°$, find how far up the ladder the man can now climb.
   (f) If, in question (d), contact is rough at both ends of the ladder and $\mu = \frac{1}{3}$ in each case, find θ if the workman can just climb to the top of the ladder. Take g as 10.

4. The foot of a uniform ladder, of length l and weight W, rests on rough horizontal ground, and the top of the ladder rests against a smooth vertical wall. The ladder is inclined at $30°$ to the vertical.
   (a) Find the magnitude of the force exerted by the ladder on the wall.
   (b) Given that the coefficient of friction between the ladder and the ground is $\frac{\sqrt{3}}{4}$, show that a person of weight $4W$ cannot climb to the top of the ladder without the ladder slipping.
   (c) Find the least weight which when placed on the foot of the ladder would enable the man to climb to the top of the ladder.

5. A uniform rod AB, of length $2l$ and weight W, is in equilibrium with the end A on a rough horizontal floor and the end B against a smooth vertical wall. The rod makes an angle $\tan^{-1} 2$ with the horizontal and is in a vertical plane which is perpendicular to the wall.
   (a) Find the least possible value of μ, the coefficient of friction between the floor and the rod.
   (b) Given that $\mu = \frac{5}{16}$, find the distance from A to the highest point of the rod at which a particle of weight W can be attached without disturbing equilibrium.

Supported beam problems

6. A uniform wooden plank AB of weight 60 kg and length 4 m crests with A on rough horizontal ground where the coefficient of friction is $\frac{1}{2}$. The plank rests in rough contact with the top C of a rail of height 1.5 m, and is just about to slip. Given that $AC = 3$ m and taking g as 10, find:
   (a) the normal contact forces at A and at C
   (b) the coefficient of friction at C

7. A uniform rod AB of length $4a$ and weight W rests with the end A in rough contact with level ground where the coefficient of friction is $\frac{1}{2}$. A point C on the rod, distant $3a$ from A, rests against a smooth peg. The rod is in limiting equilibrium when it is at $30°$ to the ground. Find in terms of W:
   (a) the normal reaction at the peg
   (b) the frictional force.

Answers

1. $\tan^{-1} \frac{2}{3} = 33.7°$
2. $\tan^{-1} 0.9 = 42.0°$
3. (a) $34°$
   (b) (i) 200 N (ii) 100 N (iii) $\frac{1}{2}$
   (c) $28°$
   (d) $R_W = 300\sqrt{3} \approx 520$N; $\mu = \frac{3\sqrt{3}}{10} \approx 0.520$
   (e) To the top
   (f) $21°$
4. (a) $\frac{W\sqrt{3}}{6}$ (b) Proof (c) W
5. (a) $\frac{1}{4}$ (b) $\frac{3}{2}l$
6. (a) $R_A = 281$ N, $R_C = 346$ N (b) $\mu_c = 0.110$
7. (a) $\frac{W\sqrt{3}}{3}$ (b) $\frac{W\sqrt{3}}{6}$