Proof by contradiction EXTRA [29]

1.

Prove by contradiction that $\sqrt[3]{2}$ is an irrational number.

[7 marks]

2.

Prove by contradiction that the difference of any rational number and any irrational number is irrational.

[4 marks]

3.

Given that a and b are distinct positive numbers, use proof by contradiction to prove that

$$\frac{a}{b}+\frac{b}{a}>2$$

[3 marks]

4.

Prove by contradiction that there are no positive integers p and q such that

$$4p^2 - q^2 = 25$$

(4)

5.

. (i) Use proof by exhaustion to show that for $n \in \mathbb{N}, n \leqslant 4$

$$(n+1)^3 > 3^n$$

(2)

(ii) Given that $m^3 + 5$ is odd, use proof by contradiction to show, using algebra, that m is even.

(4)

6.

(i) Given that p and q are integers such that

pq is even

use algebra to prove by contradiction that at least one of p or q is even.

(3)

(ii) Given that x and y are integers such that

- $x < 0$
- $(x+y)^2 < 9x^2 + y^2$

show that $y > 4x$

(2)