Proof by contradiction EXTRA MS [29]

1.

Begins proof by contradiction, assumes that $\sqrt[3]{2}$ is rational OE	AO3.1a	M1	Assume $\sqrt[3]{2}$ is rational
Uses language and notation correctly to state initial assumptions	AO2.5	B1	$\sqrt[3]{2}=\frac{a}{b}$, a and b have no common factors
Manipulates fraction including cubing.	AO1.1a	M1	$\Rightarrow \sqrt[3]{2}b=a$ $\Rightarrow 2b^3=a^3$
Deduces a is even	AO2.2a	R1	$\therefore$ a is even
Deduces b is even	AO2.2a	R1	let $a=2d$ then $2b^3=8d^3$ $\Rightarrow b^3=4d^3$ $\therefore$ b is even
Explains why there is a contradiction	AO2.4	E1	Hence, a and b have a common factor of 2. This is a contradiction.
Completes rigorous argument to show that $\sqrt[3]{2}$ is irrational	AO2.1	R1	$\therefore$ the assumption that $\sqrt[3]{2}$ is rational must be incorrect and it is proved that $\sqrt[3]{2}$ is an irrational number

2.

States assumption to begin proof by contradiction may PI by $\frac{a}{b}-x=\frac{c}{d}$ or $x-\frac{a}{b}=\frac{c}{d}$	3.1a	M1	Assume that the difference between a rational and an irrational number is rational. $\frac{a}{b}-x=\frac{c}{d}$
Uses language and notation correctly to state initial assumptions: States their a,b,c and d are integers **and** x is irrational do not accept the irrational written as a fraction Condone missing $b,d\neq 0$	2.5	A1	Where a,b,c and d are integers, $b,d\neq 0$ and x is irrational
Demonstrates that x can be expressed as a rational number by obtaining $x=\frac{ad-cb}{bd}$ OE	1.1b	M1	$x=\frac{a}{b}-\frac{c}{d}$ $=\frac{ad}{bd}-\frac{cb}{bd}$ $=\frac{ad-cb}{bd}$
Completes rigorous argument to prove the required result, clearly explaining where the contradiction lies with **ALL** assumptions correct at the start (including $b,d\neq 0$)	2.1	R1	Hence x is rational. This is a contradiction hence the difference of any rational number and any irrational number is irrational.

3.

Forms the inequality $\frac{a}{b}+\frac{b}{a}\le 2$ (for a pair of distinct positive integers a and b) Condone $\frac{a}{b}+\frac{b}{a}<2$	2.1	M1	Assume $\frac{a}{b}+\frac{b}{a}\le 2$ $\frac{a^2+b^2}{ab}\le 2$
Rearranges and factorises to deduce $(a-b)^2\le 0$ Condone $(a-b)^2<0$	2.2a	A1	$a^2+b^2\le 2ab$ $a^2-2ab+b^2\le 0$ $(a-b)^2\le 0$
Completes a reasoned argument to explain the contradiction. Must have started with $\frac{a}{b}+\frac{b}{a}\le 2$ and stated $a\ne b$ or makes reference to them being distinct.	2.1	R1	Since $a\ne b$ this is a contradiction because $(a-b)^2>0$ Hence $\frac{a}{b}+\frac{b}{a}>2$

4.

Sets up the contradiction and factorises: There are positive integers p and q such that $(2p+q)(2p-q)=25$	M1
If true then $2p+q=25$, $2p-q=1$ or $2p+q=5$, $2p-q=5$ **Award for deducing either of the above statements**	M1
Solutions are $p=6.5, q=12$ or $p=2.5, q=0$ Award for one of these	A1
This is a contradiction as there are no integer solutions hence there are no positive integers p and q such that $4p^2-q^2=25$	A1
	(4)

5.

15(i)	$n=1,\ 2^3=8,\ 3^1=3,\ (8>3)$ $n=2,\ 3^3=27,\ 3^2=9,\ (27>9)$ $n=3,\ 4^3=64,\ 3^3=27,\ (64>27)$ $n=4,\ 5^3=125,\ 3^4=81,\ (125>81)$	M1	2.1
	So if $n \leqslant 4, n \in \mathbb{N}$ then $(n+1)^3 > 3^n$	A1	2.4
		(2)	
(ii)	Begins the proof by negating the statement. "Let m be odd " or "Assume m is not even"	M1	2.4
	Set $m=(2p\pm1)$ and attempt $m^3+5=(2p\pm1)^3+5=...$	M1	2.1
	$=8p^3+12p^2+6p+6$ AND deduces even	A1	2.2a
	Completes proof which requires reason and conclusion • reason for $8p^3+12p^2+6p+6$ being even • acceptable statement such as "this is a contradiction so if m^3+5 is odd then m must be even"	A1	2.4
		(4)	
			(6 marks)

6.

7 (i)	For setting up the contradiction: There exists integers p and q such that pq is even and both p and q are odd	B1	2.5
	For example, sets $p=2m+1$ and $q=2n+1$ and then attempts $pq=(2m+1)(2n+1)=...$	M1	1.1b
	Obtains $pq=(2m+1)(2n+1)=4mn+2m+2n+1$ $=2(2mn+m+n)+1$ States that this is odd, giving a contradiction so " if pq is even, then at least one of p and q is even" *	A1*	2.1
		(3)	
(ii)			
	$(x+y)^2<9x^2+y^2 \Rightarrow 2xy<8x^2$	M1	2.2a
	States that as $x<0 \Rightarrow 2y>8x$ $\Rightarrow y>4x$ *	A1*	2.1
		(2)	
			(5 marks)