Topic X5 Numerical methods (Post-TT) [45]

1.

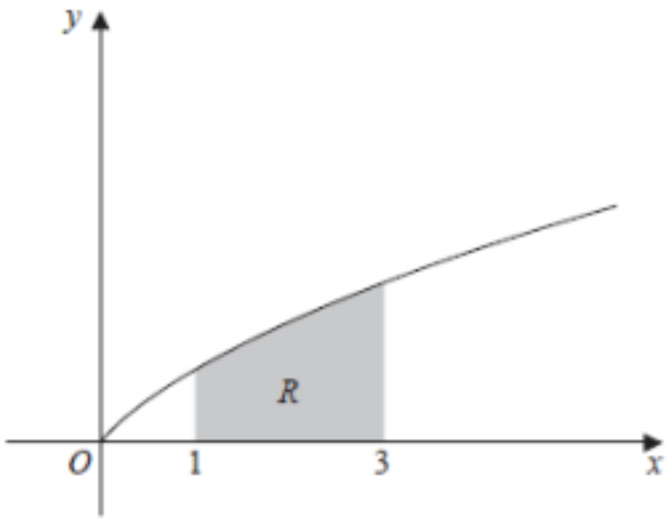

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \frac{x}{1+\sqrt{x}}$, $x \geqslant 0$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the line with equation $x = 1$, the x-axis and the line with equation $x = 3$

The table below shows corresponding values of x and y for $y = \frac{x}{1+\sqrt{x}}$

x	1	1.5	2	2.5	3
y	0.5	0.6742	0.8284	0.9686	1.0981

(a) Use the trapezium rule, with all the values of y in the table, to find an estimate for the area of R, giving your answer to 3 decimal places.

(3)

(b) Explain how the trapezium rule can be used to give a better approximation for the area of R.

(1)

(c) Giving your answer to 3 decimal places in each case, use your answer to part (a) to deduce an estimate for

(i) $\int_1^3 \frac{5x}{1+\sqrt{x}}\,dx$

(ii) $\int_1^3 \left(6 + \frac{x}{1+\sqrt{x}}\right)dx$

(2)

2.

(i) Show by means of suitable sketch graphs that the equation

$$(x-2)^4 = x + 16$$

has exactly 2 real roots. **[3]**

(ii) State the value of the smaller root. **[1]**

(iii) Use the iterative formula

$$x_{n+1} = 2 + \sqrt[4]{x_n + 16},$$

with a suitable starting value, to find the larger root correct to 3 decimal places. **[4]**

3.

The equation $x^3 - x^2 - 5x + 10 = 0$ has exactly one real root α.

(i) Show that the Newton-Raphson iterative formula for finding this root can be written as

$$x_{n+1} = \frac{2x_n^{\ 3} - x_n^{\ 2} - 10}{3x_n^{\ 2} - 2x_n - 5}.$$

[3]

(ii) Apply the iterative formula in part **(i)** with initial value $x_1 = -3$ to find x_2, x_3, x_4 correct to 4 significant figures. **[1]**

(iii) Use a change of sign method to show that $\alpha = -2.533$ is correct to 4 significant figures. **[3]**

(iv) Explain why the Newton-Raphson method with initial value $x_1 = -1$ would not converge to α. **[2]**

4.

(i) By sketching the curves $y = \ln x$ and $y = 8 - 2x^2$ on a single diagram, show that the equation

$$\ln x = 8 - 2x^2$$

has exactly one real root. **[3]**

(ii) Explain how your diagram shows that the root is between 1 and 2. **[1]**

(iii) Use the iterative formula

$$x_{n+1} = \sqrt{4 - \tfrac{1}{2}\ln x_n},$$

with a suitable starting value, to find the root. Show all your working and give the root correct to 3 decimal places. **[4]**

(iv) The curves $y = \ln x$ and $y = 8 - 2x^2$ are each translated by 2 units in the positive x-direction and then stretched by scale factor 4 in the y-direction. Find the coordinates of the point where the new curves intersect, giving each coordinate correct to 2 decimal places. **[3]**

5.

(i) Sketch the curve $y = \left(\frac{1}{2}\right)^x$, and state the coordinates of any point where the curve crosses an axis. [3]

(ii) Use the trapezium rule, with 4 strips of width 0.5, to estimate the area of the region bounded by the curve $y = \left(\frac{1}{2}\right)^x$, the axes, and the line $x = 2$. [4]

(iii) The point P on the curve $y = \left(\frac{1}{2}\right)^x$ has y-coordinate equal to $\frac{1}{6}$. Prove that the x-coordinate of P may be written as

$$1 + \frac{\log_{10} 3}{\log_{10} 2}.$$ [4]