Topic X5 Numerical methods (Post-TT) [45] MARKSCHEME

1.

1 (a)	$\text{Area}(R) \approx \frac{1}{2} \times 0.5 \times \left[0.5 + 2\left(0.6742 + 0.8284 + 0.9686\right) + 1.0981\right]$	B1	1.1b
		M1	1.1b
	$\left\{= \frac{1}{4} \times 6.5405 \;\; = \;\; 1.635125\right\} = 1.635$ (3 dp)	A1	1.1b
		(3)	
(b)	Any valid reason, for example • Increase the number of strips • Decrease the width of the strips • Use more trapezia between $x = 1$ and $x = 3$	B1	2.4
		(1)	
(c)(i)	$\left\{\int_1^3 \frac{5x}{1+\sqrt{x}}\,\mathrm{d}x\right\} = 5(\text{"1.635"}) = 8.175$	B1ft	2.2a
(c)(ii)	$\left\{\int_1^3 \left(6 + \frac{x}{1+\sqrt{x}}\right)\mathrm{d}x\right\} = 6(2) + (\text{"1.635"}) = 13.635$	B1ft	2.2a
		(2)	

2.

(i)	Draw sketch of $y = (x-2)^4$	*B1		touching positive x-axis and extending at least as far as the y-axis; no need for 2 or 16 to be marked; ignore wrong intercepts
	Draw straight line with positive gradient	*B1		at least in first quadrant and reaching positive y-axis; assess the two graphs independently of each other
	Indicate two roots	B1	**3**	AG; dep *B *B and two correct graphs which meet on the y-axis; indicated in words or by marks on sketch
	[SC: Draw sketch of $y = (x-2)^4 - x - 16$ and indicate the two roots : B1 (i.e. max 1 mark)]			
(ii)	State 0 or $x = 0$	B1	**1**	not merely for coordinates (0, 16)
(iii)	Obtain correct first iterate	B1		to at least 3 dp; any starting value (> -16)
	Show correct iteration process	M1		producing at least 3 iterates in all; may be implied by plausible converging values
	Obtain at least 3 correct iterates	A1		allowing recovery after error; iterates given to only 3 d.p. acceptable; values may be rounded or truncated
	Obtain 4.118	A1	**4**	answer required to exactly 3 dp; A0 here if number of iterates is not enough to justify 4.118; attempt consisting of answer only earns 0/4

[0 → 4 → 4.114743 → 4.117769 → 4.117849;
1 → 4.030543 → 4.115549 → 4.117790 → 4.117849;
2 → 4.059767 → 4.116321 → 4.117811 → 4.117850;
3 → 4.087798 → 4.117060 → 4.117830 → 4.117850;
4 → 4.114743 → 4.117769 → 4.117849 → 4.117851;
5 → 4.140695 → 4.118452 → 4.117867 → 4.117851]

8

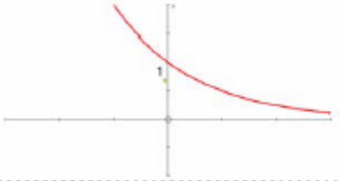
3.

(i)		$f'(x)=3x^2-2x-5$ $x_{n+1}=x_n-\dfrac{x_n^3-x_n^2-5x_n+10}{3x_n^2-2x_n-5}$ $x_{n+1}=\dfrac{3x_n^3-2x_n^2-5x_n-(x_n^3-x_n^2-5x_n+10)}{3x_n^2-2x_n-5}=$ $=\dfrac{2x_n^3-x_n^2-10}{3x_n^2-2x_n-5}$	B1 M1 E1 [3]	1.1 1.1 2.1	Substitute into correct formula for Newton-Raphson AG a correct intermediate step leading to the given answer is required	
(ii)		$x_2=-2.607$ $x_3=-2.535$ $x_4=-2.533$	B1 [1]	1.1	BC All three values must be given to 4 significant figures.	
(iii)		$f(-2.5325)$ and $f(-2.5335)$ $(-2.5325)^3-(-2.5325)^2-5(-2.5325)+10=$ 0.0066125 $(-2.5335)^3-(-2.5335)^2-5(-2.5335)+10=$ −0.0127017 Since $f(-2.5325)>0$ and $f(-2.5335)<0$ x_4 is α to 4 s.f.	M1 A1 E1 [3]	1.1 2.1 2.4	Accept other alternative values which would confirm α as a root correct to 4 s.f. At least the result of evaluation must be shown The change of sign must be pointed to	
(iv)		$3(-1)^2-2(-1)-5=0$ Since the fraction is undefined at $x=-1$, x_2 is undefined	B1 E1 [2]	2.1 1.2	Accept references to a stationary point of the function	or the tangent to the curve being horizontal

4.

(i)		Sketch more or less correct $y=\ln x$ Sketch more or less correct $y=8-2x^2$ Indicate intersection by some mark on diagram (just a 'blob' sufficient) of by statement in words away from diagram	B1 B1 B1 [3]	existing for positive and negative y; no need to indicate (1, 0); ignore any scales given on axes; condone graph touching y-axis but B0 if it crosses y-axis (roughly) symmetrical about y-axis; extending, if minimally, into quadrants for which $y<0$; no need to indicate $(\pm 2, 0)$, (0, 8); assess each curve separately needs each curve to be (more or less) correct in the first quadrant and on curves being related to each other correctly there
(ii)		Refer, in some way, to graphs crossing x-axis at $x=1$ and $x=2$ and that intersection is between these values	B1 [1]	AG; the values 1 and 2 may be assumed from part (i) if clearly marked there; dependent on curves being (more or less) correct in first quadrant; carrying out the sign-change routine is B0
(iii)		Obtain correct first iterate Show correct iterative process Obtain at least 3 correct iterates Conclude with 1.917 1 → 2 → 1.91139 → 1.91731... → 1.91690... → 1.91693... 1.5 → 1.94865... → 1.91479... → 1.91707... → 1.91692... 2 → 1.91139... → 1.91731... → 1.91690... → 1.91693...	B1 M1 A1 A1 [4]	to at least 3 dp (except in the case of starting value 1 leading to 2) involving at least 3 iterates in all; may be implied by plausible converging values allowing recovery after error; iterates given to at least 3 dp; values may be rounded or truncated answer required to exactly 3 dp; answer only with no evidence of process is 0/4
(iv)		Obtain 3.92 or greater accuracy Attempt $4\times$ ln(part (iii) answer) Obtain y-coordinate 2.60	B1√ M1 A1 [3]	following their answer to part (iii) value required to exactly 2 dp (so A0 for 2.6 and 2.603)

5.

(i)		M1 A1 B1	3	Attempt sketch of any exponential graph, in at least first quadrant Correct graph – must be in both quadrants For identification of (0, 1)
(ii)	$A \approx \frac{1}{2} \times 0.5 \times \left\{1 + 2\left(0.5^{\frac{1}{2}} + 0.5 + 0.5^{\frac{3}{2}}\right) + 0.5^2\right\}$ ≈ 1.09	B1 M1 A1 A1	4	State, or imply, at least three correct *y*-values For correct use of trapezium rule, inc correct *h* For correct unsimplified expression For the correct value 1.09, or better
(iii)	$\left(\frac{1}{2}\right)^x = \frac{1}{6} \Rightarrow x\log_{10}\frac{1}{2} = \log_{10}\frac{1}{6}$ $x = \frac{\log_{10}\frac{1}{6}}{\log_{10}\frac{1}{2}} = \frac{-\log_{10}6}{-\log_{10}2}$ Hence $= \frac{\log_{10}2 + \log_{10}3}{\log_{10}2}$ $= 1 + \frac{\log_{10}3}{\log_{10}2}$	M1 A1 M1 A1	4	For equation $\left(\frac{1}{2}\right)^x = \frac{1}{6}$ and attempt at logs Obtain $x\log\left(\frac{1}{2}\right) = \log\left(\frac{1}{6}\right)$, or equivalent For use of log 6 = log 2 + log 3 For showing the given answer correctly
	OR $\left(\frac{1}{2}\right)^x = \frac{1}{6} \Rightarrow 2^x = 6$ $\Rightarrow x\log_{10}2 = \log_{10}6$ $x = \frac{\log_{10}6}{\log_{10}2}$ $= \frac{\log_{10}2 + \log_{10}3}{\log_{10}2}$ $= 1 + \frac{\log_{10}3}{\log_{10}2}$	M1 A1 M1 A1		For equation $2^x = 6$ and attempt at logs Obtain *x* log 2 = log 6, or equivalent For use of log 6 = log 2 + log 3 For showing the given answer correctly
	OR $\left(\frac{1}{2}\right)^x = \frac{1}{6} \Rightarrow 2^x = 6$ $2^{x-1} = 3$ $(x-1)\log_{10}2 = \log_{10}3$ Hence $x = 1 + \frac{\log_{10}3}{\log_{10}2}$	M1 A1 M1 A1		Attempt to rearrange equation to $2^n = 3$ Obtain $2^{x-1} = 3$ For attempt at logs For showing the given answer correctly
	OR $x = \frac{\log_{10}2 + \log_{10}3}{\log_{10}2}$ $= \frac{\log_{10}6}{\log_{10}2}$ $x\log_{10}2 = \log_{10}6$ $\log_{10}2^x = \log_{10}6$ $2^x = 6$ $\left(\frac{1}{2}\right)^x = \frac{1}{6}$	M1 A1 M1 A1		Use log 2 + log 3 = log 6 Obtain *x*log 2 = log 6 Attempt to remove logarithms Show $\left(\frac{1}{2}\right)^x = \frac{1}{6}$ correctly
			11	