Topic X5 Numerical methods (Pre-TT) [42]

1.

(i) It is given that k is a positive constant. By sketching the graphs of

$$y = 14 - x^2 \text{ and } y = k\ln x$$

on a single diagram, show that the equation

$$14 - x^2 = k\ln x$$

has exactly one real root. **[3]**

(ii) The real root of the equation $14 - x^2 = 3\ln x$ is denoted by α.

(a) Find by calculation the pair of consecutive integers between which α lies. **[3]**

(b) Use the iterative formula $x_{n+1} = \sqrt{14 - 3\ln x_n}$, with a suitable starting value, to find α. Show the result of each iteration, and give α correct to 2 decimal places. **[4]**

(Total 10 marks)

2.

(i) Use the trapezium rule, with 4 strips each of width 0.5, to find an approximate value for

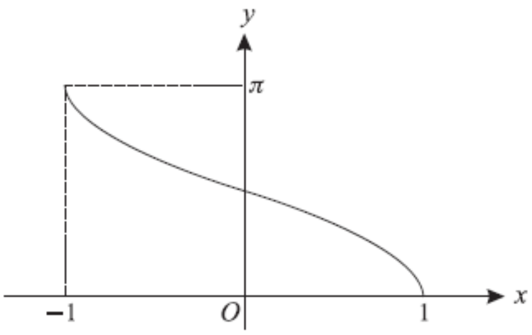
$$\int_3^5 \log_{10}(2 + x)\,\mathrm{d}x,$$

giving your answer correct to 3 significant figures. **[4]**

(ii) Use your answer to part **(i)** to deduce an approximate value for $\int_3^5 \log_{10}\sqrt{2+x}\,\mathrm{d}x$, showing your method clearly. **[2]**

(Total 6 marks)

3.



The diagram shows the curve with equation $y = \cos^{-1} x$.

(i) Sketch the curve with equation $y = 3\cos^{-1}(x-1)$, showing the coordinates of the points where the curve meets the axes. [3]

(ii) By drawing an appropriate straight line on your sketch in part **(i)**, show that the equation $3\cos^{-1}(x-1) = x$ has exactly one root. [1]

(iii) Show by calculation that the root of the equation $3\cos^{-1}(x-1) = x$ lies between 1.8 and 1.9. [2]

(iv) The sequence defined by

$$x_1 = 2, \qquad x_{n+1} = 1 + \cos\left(\tfrac{1}{3}x_n\right)$$

converges to a number α. Find the value of α correct to 2 decimal places and explain why α is the root of the equation $3\cos^{-1}(x-1) = x$. [5]

(Total 11 marks)

4.

The equation $x^3 - 3x + 1 = 0$ has three real roots.

(a) Show that one of the roots lies between −2 and −1

[2 marks]

(b) Taking $x_1 = -2$ as the first approximation to one of the roots, use the Newton-Raphson method to find x_2, the second approximation.

[3 marks]

(c) Explain why the Newton-Raphson method fails in the case when the first approximation is $x_1 = -1$

[1 mark]

(Total 6 marks)

5.

(i) Use the trapezium rule, with 2 strips each of width 4, to show that an approximate value of $\int_1^9 4\sqrt{x}\,dx$ is $32 + 16\sqrt{5}$. **[3]**

(ii) Use a sketch graph to explain why the actual value of $\int_1^9 4\sqrt{x}\,dx$ is greater than $32 + 16\sqrt{5}$. **[2]**

(iii) Use integration to find the exact value of $\int_1^9 4\sqrt{x}\,dx$. **[4]**

(Total 9 marks)