Topic X5 Numerical methods (Pre-TT) [42] MARKSCHEME

1.

(i)		Sketch (more or less) correct $y = 14 - x^2$	B1	assessed separately from other graph; must exist in all four quadrants; ignore any intercepts given	
		Sketch (more or less) correct $y = k\ln x$	B1	assessed separately from other graph; must exist in first and fourth quadrants; if clearly meets y-axis award B0; if clear maximum point in first quadrant award B0	
		Indicate one root ('blob' on sketch or written reference to one intersection or …)	B1 [3]	dependent on both curves being correct in first quadrant and there being no possibility, from their graphs, of further points of intersection elsewhere	
(ii)	(a)	Calculate values for at least 2 integers	M1		
		Obtain correct values for $x = 3$ and $x = 4$	A1	$14 - x^2 - 3\ln x$: 1.7 −6.2 $14 - x^2$, $3\ln x$: 5, 3.3 −2, 4.2	
		State 3 and 4	A1 [3]	following correct calculations	
(ii)	(b)	Obtain correct first iterate	B1	having started with any positive value; B1 available if 'iteration' never goes beyond a first iterate;	
		Attempt iteration process	M1	implied by plausible sequence of values	
		Obtain at least 3 correct iterates in all	A1	showing at least 2 d.p.	
		Obtain 3.24	A1 [4]	answer required to exactly 2 d.p; not given for 3.24 as the final iterate in a sequence, i.e. needs an indication (perhaps just underlining) that value of α found [3 → 3.27172 → 3.23173 → 3.23743 → 3.23661 3.5 → 3.20027 → 3.24196 → 3.23596 → 3.23682 4 → 3.13706 → 3.25118 → 3.23465 → 3.23701]	

2.

(i)	$\int_3^5 \log_{10}(2+x)\,dx \approx \frac{1}{2}\times\frac{1}{2}\times(\log 5 + 2\log 5.5 +$	M1		Attempt y-coords for at least 4 of the correct 5 x-coords only
	$2\log 6 + 2\log 6.5 + \log 7)$	M1		Use correct trapezium rule, any h, to find area between $x = 3$ and $x = 5$
		M1		Correct h (soi) for their y-values
	≈ 1.55	A1	4	Obtain 1.55
(ii)	$\int_3^5 \log_{10}(2+x)^{\frac{1}{2}}\,dx = \frac{1}{2}\int_3^5 \log_{10}(2+x)\,dx$	B1√		Divide by 2, or equiv, at any stage to obtain 0.78 or 0.77,
	$\approx ½ \times 1.55$			following their answer to **(i)**
	≈ 0.78	B1	2	Explicitly use log $\sqrt{a}$ = ½ log a on a single term
			6	

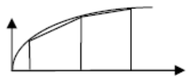
3.

(i)	Sketch curve showing (at least) translation in x direction	**M1**		[either positive or negative]
	Show correct sketch with one of 2 and 3π indicated	**A1**		
	… and with other one of 2 and 3π indicated	**A1**	**3**	
(ii)	Draw straight line through O with positive gradient	**B1**	**1**	[label and explanation not required]
(iii)	Attempt calculations using 1.8 and 1.9	**M1**		[allow here if degrees used]
	Obtain correct values and indicate change of sign	**A1**	**2**	[or equiv; $x = 1.8$: LHS = 1.93, diff = 0.13; $x = 1.9$: LHS = 1.35, diff = -0.55; radians needed now]
(iv)	Obtain correct first iterate 1.79 or 1.78	**B1**		[or greater accuracy]
	Attempt correct process to produce at least 3 iterates	**M1**		
	Obtain 1.82	**A1**		[answer required to exactly 2 d.p.; $2 \to 1.7859 \to 1.8280 \to 1.8200$; SR: answer 1.82 only - B2]
	Attempt rearrangement of $3\cos^{-1}(x-1) = x$ or of $x = 1 + \cos(\frac{1}{3}x)$	**M1**		[involving at least two steps]
	Obtain required formula or equation respectively	**A1**	**5**	

4.

a)	Starts an argument by showing that $f(-2) < 0$ and $f(-1) > 0$ Both attempted and at least one evaluated correctly f must be clearly defined or substitution of values must be explicit.	AO2.1	R1	$f(x) = x^3 - 3x + 1$ $f(-2) = (-2)^3 - 3(-2) + 1 = -1 < 0$ $f(-1) = (-1)^3 - 3(-1) + 1 = 3 > 0$ Change of sign and $f(x)$ is continuous so a root must lie between $x = -2$ and $x = -1$
	Explains reasoning fully to complete the argument Evaluations above need to be of opposite sign **and** 'change of sign' OE seen **and** reference to x-values –2 & –1 **and** reference to continuous function	AO2.4	E1	
(b)	Uses Newton–Raphson, must have $f'(x)$ correct PI by correct substitution	AO1.1a	M1	$x_{n+1} = x_n - \dfrac{(x_n)^3 - 3x_n + 1}{3(x_n)^2 - 3}$
	Substitutes $x_1 = -2$ into 'their' Newton–Raphson formula (accept 'their' $f(-2)$ from part **(a)**)	AO1.1a	M1	$x_2 = -2 - \dfrac{-1}{3(-2)^2 - 3}$
	Obtains correct value for x_2 $-\dfrac{17}{9}$ or $-1\dfrac{8}{9}$ or -1.89 or better	AO1.1b	A1	$= -\dfrac{17}{9}$
(c)	Explains why the method fails when $x_1 = -1$ This must include a substitution of $x_1 = -1$ and an explanation of what goes wrong eg division by zero | not possible gradient zero | method fails	AO2.4	E1	$x_1 = -1$ $x_2 = -2 = \dfrac{3}{3(-1)^2 - 3} = -2 - \dfrac{3}{0}$ causes division by zero (expression undefined) **ALT** $f'(-1) = 0$, function has zero gradient at this point, method will fail
	Total		**6**	

5.

(i)		$0.5 \times 4 \times (4\sqrt{1} + 8\sqrt{5} + 4\sqrt{9})$	M1*	Attempt y-values at $x = 1, 5, 9$ only	Must be using y, not an attempt at integration. Allow slips eg $\sqrt{(4x)}$ as long as clearly intended as y. Allow decimal equiv for y_1 (8.94). Allow M1 for 4, 20, 72 (ie omitting the $\sqrt{\ }$). M0 if other y-values found (unless not used in trap rule).
		$= 2(16 + 8\sqrt{5})$	M1d*	Attempt correct trapezium rule, inc $h = 4$	Correct structure, including 'big brackets' seen or implied. Allow 2 used for ½ h – no need for ½ x 4 to be explicit. Allow slips when calculating y values, but all other aspects must be correct. Could use two separate trapezia.
		$= 32 + 16\sqrt{5}$ **AG**	A1 **[3]**	Obtain $32 + 16\sqrt{5}$	Must come from exact working, so A0 if answer first found in decimals (67.777…) which is then stated to be the same as $32 + 16\sqrt{5}$. However, isw if exact answer found first, and then decimal equiv stated.
(ii)			B1*	Sketch showing correct graph of $y = 4\sqrt{x}$ and two trapezia (allow if only tops of trapezia seen as chords)	Correct graph shown, existing for at least $1 \le x \le 9$. Exactly two trapezia must be shown, of roughly equal widths, with top vertices on the curve.
		Curve is above tops of trapezia	B1d* **[2]**	Reason comparing the tops of trapezia to the curve, or referring to the gap between the trapezia and the curve	Must refer to the tops of the trapezia so B0 for 'trapezia are below curve' (ie 'top' not used). Allow 'trapezium' rather than 'trapezia'. Could shade gaps on their diagram but some text also reqd. B0 for 'some area not calculated' unless clear which area. Concave / convex is B0, as is comparing to exact area. B1 for decreasing gradient (but B0 for decreasing curve). B0 (rather than isw) if explanation is partially incorrect. No sketch is B0, irrespective of explanation given. **SR B1** for correct explanation, and trapezia, and correct graph of $y = 4\sqrt{x}$ for $1 \le x \le 9$ but incorrect outside range (eg curvature / y-intercept / not just in first quadrant).
(iii)		$\int_1^9 4x^{\frac{1}{2}}\,dx = \left[\frac{8}{3}x^{\frac{3}{2}}\right]_1^9$	M1	Obtain $kx^{\frac{3}{2}}$	Any numerical k, including 4. Any exact equiv for the index.
		$= 72 - \frac{8}{3}$	A1	Obtain $\frac{8}{3}x^{\frac{3}{2}}$	Allow unsimplified coefficient, inc $\frac{4}{1.5}$ or $\frac{2}{3} \times 4$. Allow non exact decimal ie 2.7, 2.67 etc. Allow $+ c$.
		$= 69\frac{1}{3}$	M1	Attempt correct use of limits	Must be F(9) – F(1) ie subtraction with limits in the correct order. Allow use in any function other than the original, including from differentiation. Allow processing errors eg $\left(\frac{8}{3} \times 9\right)^{1.5}$.
			A1 **[4]**	Obtain $69\frac{1}{3}$, or any exact equiv	Allow improper fraction, or recurring decimal. A0 for 69.333…. A0 for $69\frac{1}{3} + c$. Answer only is 0/4.