

Topic X6 Further differentiation (Post-TT) [43]

1.

Show that $\int_{\sqrt{2}}^{\sqrt{6}} \frac{2}{x} dx = \ln 3$.

[3]

(Total 3 marks)

2.

(a) Differentiate $x^2(x+1)^6$ with respect to x .

[3]

(b) Find the gradient of the curve $y = \frac{x^2 + 3}{x^2 - 3}$ at the point where $x = 1$.

[3]

(Total 6 marks)

3.

The equation of a curve is $x^2 + 3xy + 4y^2 = 58$. Find the equation of the normal at the point (2, 3) on the curve, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

[8]

(Total 8 marks)

4.

Find, in the form $y = mx + c$, the equation of the tangent to the curve

$$y = x^2 \ln x$$

at the point with x -coordinate e .

[6]

(Total 6 marks)

5.

(a) Show that, for all non-zero values of the constant k , the curve

$$y = \frac{kx^2 - 1}{kx^2 + 1}$$

has exactly one stationary point.

[5]

(b) Show that, for all non-zero values of the constant m , the curve

$$y = e^{mx}(x^2 + mx)$$

has exactly two stationary points.

[7]

(Total 12 marks)

6.

The equation of a curve is $2x^2 + xy + y^2 = 14$. Show that there are two stationary points on the curve and find their coordinates.

[8]

(Total 8 marks)