Topic X6 Further differentiation (Post-TT) [43] MARKSCHEME

1.

		State $2\ln x$ Use both relevant logarithm properties correctly Obtain $\ln 3$	B1 M1 A1 [3]	may be implied by immediate use of limits either or both may be implied, eg by $2\ln\sqrt{6} = \ln 6$ or by $\ln 6 - \ln 2 = \ln 3$ AG; with at least one property shown explicitly

2.

(a)	Attempt use of product rule	**M1**		[involving … + …]
	Obtain $2x(x+1)^6$ …	**A1**		
	Obtain … + $6x^2(x+1)^5$	**A1**	**3**	[or equivs; ignore subsequent attempt at simplification]
(b)	Attempt use of quotient rule	**M1**		[or, with adjustment, product rule; allow u / v confusion]
	Obtain $\dfrac{(x^2-3)2x-(x^2+3)2x}{(x^2-3)^2}$	**A1**		[or equiv]
	Obtain -3	**A1**	**3**	[from correct derivative only]

3.

(i) $\dfrac{d}{dx}(y^2) = 2y\dfrac{dy}{dx}$	B1		
Using $d(uv) = u\,dv + v\,du$ for the $(3)xy$ term	M1		
$\dfrac{d}{dx}(x^2+3xy+4y^2) = 2x + 3x\dfrac{dy}{dx} + 3y + 8y\dfrac{dy}{dx}$	A1		
Solve for $\dfrac{dy}{dx}$ & subst $(x, y) = (2,3)$	M1		or v.v. Subst now or at normal eqn stage; (M1 dep on either/both B1 M1 earned)
$\dfrac{dy}{dx} = -\dfrac{13}{30}$	A1		Implied if grad normal = $\dfrac{30}{13}$
Grad normal = $\dfrac{30}{13}$ follow-through	√B1		This f.t. mark awarded only if numerical
Find equ <u>any</u> line thro (2,3) with <u>any</u> num grad	M1		
$30x - 13y - 21 = 0$ AEF	A1	8	No fractions in final answer **8**

4.

Attempt use of product rule	**M1**	… + … form
Obtain $2x\ln x + x^2 . \dfrac{1}{x}$	**A1**	or equiv
Substitute e to obtain 3e for gradient	**A1**	or exact (unsimplified) equiv
Attempt eqn of straight line with numerical gradient	**M1**	allowing approx values
Obtain $y - e^2 = 3e(x - e)$	**A1**√	or equiv; following their gradient provided obtained by diffn attempt; allow approx values
Obtain $y = 3ex - 2e^2$	**A1**	in terms of e now and in requested form
	6	

5.

(a)	Attempt use of quotient rule	*M1		or equiv; allow numerator wrong way round and denominator errors
	Obtain $\frac{(kx^2+1)2kx-(kx^2-1)2kx}{(kx^2+1)^2}$	A1		or equiv; with absent brackets implied by subsequent correct working
	Obtain correct simplified numerator $4kx$	A1		
	Equate numerator of first derivative to zero	M1		dep *M
	State $x=0$ or refer to $4kx$ being linear or observe that, with $k \neq 0$, only one sol'n	A1√	5	AG or equiv; following numerator of form $k'kx=0$, any constant k'
(b)	Attempt use of product rule	*M1		
	Obtain $me^{mx}(x^2+mx)+e^{mx}(2x+m)$	A1		or equiv
	Equate to zero and either factorise with factor e^{mx} or divide through by e^{mx}	M1		dep *M
	Obtain $mx^2+(m^2+2)x+m=0$ or equiv and observe that e^{mx} cannot be zero	A1		
	Attempt use of discriminant	M1		using correct b^2-4ac with their a, b, c
	Simplify to obtain m^4+4	A1		or equiv
	Observe that this is positive for all m and hence two roots	A1	7	or equiv; AG
			12	

6.

$\frac{d}{dx}(xy)=x\frac{dy}{dx}+y$	B1		
$\frac{d}{dx}(y^2)=2y\frac{dy}{dx}$	B1		
$4x+x\frac{dy}{dx}+y+2y\frac{dy}{dx}=0$	B1		
Put $\frac{dy}{dx}=0$	*M1		
Obtain $4x+y=0$ AEF	A1		and no other (different) result
Attempt to solve simultaneously with eqn of curve	dep*M1		
Obtain $x^2=1$ or $y^2=16$ from $4x+y=0$	A1		
$(1,-4)$ and $(-1,4)$ and no other solutions	A1	8	Accept $(\pm 1, \mp 4)$ but not $(\pm 1, \pm 4)$