

Topic X6 Further differentiation (Pre-TT) [46]

1.

Differentiate each of the following with respect to x .

(i) $x^3(x+1)^5$ [2]

(ii) $\sqrt{3x^4+1}$ [3]

(Total 5 marks)

2.

Show that $\int_2^8 \frac{3}{x} dx = \ln 64$. [4]

(Total 4 marks)

3.

(a) Find the exact value of the x -coordinate of the stationary point of the curve $y = x \ln x$. [4]

(b) The equation of a curve is $y = \frac{4x+c}{4x-c}$, where c is a non-zero constant. Show by differentiation that this curve has no stationary points. [3]

(Total 7 marks)

4.

It is given that $y = 5^{x-1}$.

(i) Show that $x = 1 + \frac{\ln y}{\ln 5}$. [2]

(ii) Find an expression for $\frac{dx}{dy}$ in terms of y . [2]

(iii) Hence find the exact value of the gradient of the curve $y = 5^{x-1}$ at the point $(3, 25)$. [2]

(Total 6 marks)

5.

Find the equation of the normal to the curve

$$x^3 + 4x^2y + y^3 = 6$$

at the point $(1, 1)$, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. [6]

(Total 6 marks)

6.

A curve has equation $y = \frac{xe^{2x}}{x+k}$, where k is a non-zero constant.

(i) Differentiate xe^{2x} , and show that $\frac{dy}{dx} = \frac{e^{2x}(2x^2 + 2kx + k)}{(x+k)^2}$. [5]

(ii) Given that the curve has exactly one stationary point, find the value of k , and determine the exact coordinates of the stationary point. [5]

(Total 10 marks)

7.

A sculpture formed from a prism is fixed on a horizontal platform, as shown in the diagram.

The shape of the cross-section of the sculpture can be modelled by the equation $x^2 + 2xy + 2y^2 = 10$, where x and y are measured in metres.

The x and y axes are horizontal and vertical respectively.



Find the maximum vertical height above the platform of the sculpture.

[8 marks]

(Total 8 marks)