Topic X6 Further differentiation (Pre-TT) [46] MARKSCHEME

1.

(i)	Attempt use of product rule	M1		
	Obtain $3x^2(x+1)^5 + 5x^3(x+1)^4$	A1	2	or equiv
	[Or: (following complete expansion and differentiation term by term)			
	Obtain $8x^7 + 35x^6 + 60x^5 + 50x^4 + 20x^3 + 3x^2$	B2		allow B1 if one term incorrect]
(ii)	Obtain derivative of form $kx^3(3x^4+1)^n$	M1		any constants k and n
	Obtain derivative of form $kx^3(3x^4+1)^{-\frac{1}{2}}$	M1		
	Obtain correct $6x^3(3x^4+1)^{-\frac{1}{2}}$	A1	3	or (unsimplified) equiv

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2.

Obtain integral of form $k\ln x$	M1		[any non-zero constant k; or equiv such as $k\ln 3x$]
Obtain $3\ln 8 - 3\ln 2$	A1		[or exact equiv]
Attempt use of at least one relevant log property	M1		[would be earned by initial $\ln x^3$]
Obtain $3\ln 4$ or $\ln 8^3 - \ln 2^3$ and hence $\ln 64$	A1	4	[AG; with no errors]

3.

(a)	Attempt use of product rule	*M1	
	Obtain $\ln x + 1$	A1	[or unsimplified equiv]
	Equate attempt at first derivative to zero and obtain value involving e	M1	[dependent on *M]
	Obtain e^{-1}	A1	4 [or exact equiv]
(b)	Attempt use of quotient rule	M1	[or equiv using product rule or …]
	Obtain $\dfrac{(4x-c)4 - 4(4x+c)}{(4x-c)^2}$	A1	[or equiv]
	Show that first derivative cannot be zero	A1	3 [AG; derivative must be correct]

4.

(i)	State $\ln y = (x-1)\ln 5$	B1		whether following $\ln y = \ln 5^{x-1}$ or not; brackets needed
	Obtain $x = 1 + \dfrac{\ln y}{\ln 5}$	B1	2	AG; correct working needed; missing brackets maybe now implied
(ii)	Differentiate to obtain single term of form $\dfrac{k}{y}$	M1		any constant k
	Obtain $\dfrac{1}{y\ln 5}$	A1	2	or equiv involving y
(iii)	Substitute for y and attempt reciprocal	M1		or equiv method for finding derivative without using part **(ii)**
	Obtain $25\ln 5$	A1	2	or exact equiv

5.

$\frac{d}{dx}(x^2y) = x^2\frac{dy}{dx} + 2xy$	B1		s.o.i.;
$\frac{d}{dx}(y^3) = 3y^2\frac{dy}{dx}$	B1		
Substitute $(x,y) = (1,1)$ and solve for $\frac{dy}{dx}$	M1		or v.v. Solve now or at normal stage. [This
	M1		dep on either/both B1 earned]
$\frac{dy}{dx} = -\frac{11}{7}$ WWW	A1		Implied if grad normal = $\frac{7}{11}$
Gradient normal = $-\frac{1}{\frac{dy}{dx}}$	M1		Numerical or general, awarded at any stage
$7x - 11y + 4 = 0$ AEF	A1	**6**	No fractions in final answer.

6.

(i)	Attempt use of product rule for xe^{2x}	**M1**		obtaining … + …
	Obtain $e^{2x} + 2xe^{2x}$	**A1**		or equiv; maybe within QR attempt
	Attempt use of quotient rule	**M1**		with or without product rule
	Obtain unsimplified $\frac{(x+k)(e^{2x} + 2xe^{2x}) - xe^{2x}}{(x+k)^2}$	**A1**		
	Obtain $\frac{e^{2x}(2x^2 + 2kx + k)}{(x+k)^2}$	**A1**	**5**	**AG**; necessary detail required
(ii)	Attempt use of discriminant	**M1**		or equiv
	Obtain $4k^2 - 8k = 0$ or equiv and hence $k = 2$	**A1**		
	Attempt solution of $2x^2 + 2kx + k = 0$	**M1**		using their numerical value of k or solving in terms of k using correct formula
	Obtain $x = -1$	**A1**		
	Obtain $-e^{-2}$	**A1**	**5**	or exact equiv

7.

Finds the difference between the maximum and minimum values of y	AO3.1b	M1	$x^2+2xy+2y^2=10$ $2x+2y+2x\frac{\mathrm{d}y}{\mathrm{d}x}+4y\frac{\mathrm{d}y}{\mathrm{d}x}=0$ Highest and lowest points occur when $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $\frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow x=-y$ $y^2-2y^2+2y^2=10$ $y=\pm\sqrt{10}$ $\therefore$Height = $\sqrt{10}-(-\sqrt{10})$ = $2\sqrt{10}$ = 6.32 m
Uses implicit differentiation	AO1.1a	M1	
Differentiates correctly	AO1.1b	A1	
States stationary points occur when $\frac{\mathrm{d}y}{\mathrm{d}x}=0$	AO2.4	R1	
Uses $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ to find x in terms of y (or vice versa)	AO1.1a	M1	
Finds $x = -y$	AO1.1b	A1	
Deduces maximum and minimum values of y FT 'their' expression provided all M1 marks have been awarded	AO2.2a	A1F	
States the height of the sculpture above the platform FT 'their' max and min values for y provided all M1 marks have been awarded	AO2.2a	A1F	
Total		**8**	