

Topic X7 Further integration (Post-TT) [33]

1.

Given that

$$\int_0^{\ln 4} (ke^{3x} + (k-2)e^{-\frac{1}{2}x}) dx = 185,$$

find the value of the constant k .

[7]

(Total 7 marks)

2.

Find the exact value of $\int_1^e x^4 \ln x dx$.

[5]

(Total 5 marks)

3.

(i) Show that the substitution $u = \sqrt{x}$ transforms $\int \frac{1}{x(1+\sqrt{x})} dx$ to $\int \frac{2}{u(1+u)} du$. [3]

(ii) Hence find the exact value of $\int_1^9 \frac{1}{x(1+\sqrt{x})} dx$. [5]

(Total 8 marks)

4.

(i) Show that the substitution $x = \sin^2 \theta$ transforms $\int \sqrt{\frac{x}{1-x}} dx$ to $\int 2 \sin^2 \theta d\theta$. [4]

(ii) Hence find $\int_0^1 \sqrt{\frac{x}{1-x}} dx$. [5]

(Total 9 marks)

5.

Find $\int x \sec^2 x dx$.

[4]

(Total 4 marks)