Topic X7 Further calculus (Post-TT A) [48]

1.

A curve has equation $y = x^2 + kx - 4x^{-1}$ where k is a constant. Given that the curve has a minimum point when $x = -2$

- find the value of k,
- show that the curve has a point of inflection which is not a stationary point.

[7]

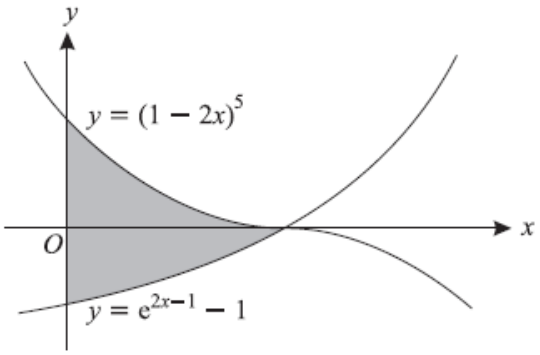
(Total 7 marks)

2.

Find the exact value of $\int_1^e x^4 \ln x \, dx$. [5]

(Total 5 marks)

3.



The diagram shows the curves $y = (1 - 2x)^5$ and $y = e^{2x-1} - 1$. The curves meet at the point $(\frac{1}{2}, 0)$. Find the exact area of the region (shaded in the diagram) bounded by the y-axis and by part of each curve. [8]

(Total 8 marks)

4.

A curve has parametric equations

$$x = t^2 - 6t + 4, \quad y = t - 3.$$

Find

(i) the coordinates of the point where the curve meets the x-axis, **[2]**

(ii) the equation of the curve in cartesian form, giving your answer in a simple form without brackets, **[2]**

(iii) the equation of the tangent to the curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[5]**

(Total 9 marks)

5.

(i) Show that the substitution $x = \sin^2\theta$ transforms $\int \sqrt{\frac{x}{1-x}}\,\mathrm{d}x$ to $\int 2\sin^2\theta\,\mathrm{d}\theta$. [4]

(ii) Hence find $\int_0^1 \sqrt{\frac{x}{1-x}}\,\mathrm{d}x$. [5]

(Total 9 marks)

6.

The height, h metres, of a shrub t years after planting is given by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{6-h}{20}.$$

A shrub is planted when its height is 1 m.

(i) Show by integration that $t = 20\ln\left(\frac{5}{6-h}\right)$. [6]

(ii) How long after planting will the shrub reach a height of 2 m? [1]

(iii) Find the height of the shrub 10 years after planting. [2]

(iv) State the maximum possible height of the shrub. [1]

(Total 10 marks)