Topic X7 Further calculus (Post-TT A) [48] MARKSCHEME

1.

$\frac{dy}{dx} = 2x + k + 4x^{-2}$	M1	1.1a	Attempt to differentiate	Power decreases by 1 for at least 2 terms
$2(-2) + k + 4(-2)^{-2} = 0$	M1	3.1a	Substitute $x = -2$, equate to 0 and attempt to solve	
$k = 3$	A1	1.1		
$\frac{d^2y}{dx^2} = 2 - 8x^{-3}$				
$2 - 8x^{-3} = 0$	M1	3.1a	Equate second derivative to 0 and attempt to solve	
$x = 4^{\frac{1}{3}}$	A1	1.1		
for $x < 4^{\frac{1}{3}} \Rightarrow \frac{d^2y}{dx^2} < 0$ for $x > 4^{\frac{1}{3}} \Rightarrow \frac{d^2y}{dx^2} > 0$	E1	2.1	Consider convex/concave either side of $x = 4^{\frac{1}{3}}$ and conclude	
When $x = 4^{\frac{1}{3}}, \frac{dy}{dx} \neq 0$ hence not a stationary point	E1	2.1	Consider gradient at $x = 4^{\frac{1}{3}}$, or justify that $x = -2$ is the only stationary point	
	[7]			

2.

Parts with correct split of $u = \ln x, \frac{dv}{dx} = x^4$	*M1	obtaining result $f(x) +/- \int g(x)dx$
$\frac{x^5}{5}\ln x - \int \frac{x^5}{5}.\frac{1}{x}(dx)$	A1	
$\frac{x^5}{5}\ln x - \frac{x^5}{25}$	A1	
Correct method with the limits	dep*M1	Decimals acceptable here
$\frac{4e^5}{25} + \frac{1}{25}$ ISW (Not '+c')	A1	Accept equiv fracts; like terms amalgamated
	5	

3.

Obtain integral of form $k(1-2x)^6$	M1		[any non-zero constant k]
Obtain correct $-\frac{1}{12}(1-2x)^6$	A1		[or unsimplified equiv; allow $+c$]
Use limits to obtain $\frac{1}{12}$	A1		[or exact (unsimplified) equiv]
Obtain integral of form ke^{2x-1}	M1		[or equiv; any non-zero constant k]
Obtain correct $\frac{1}{2}e^{2x-1} - x$	A1		[or equiv; allow $+c$]
Use limits to obtain $-\frac{1}{2}e^{-1}$	A1		[or exact (unsimplified) equiv]
Show correct process for finding required area	M1		[at any stage of solution; if process involves two definite integrals, second must be negative]
Obtain $\frac{1}{12} + \frac{1}{2}e^{-1}$	A1	8	[or exact equiv; no $+c$]

4.

(i)	Solve $0 = t-3$ & subst into $x = t^2 - 6t + 4$	M1	
	Obtain $x = -5$	A1 **(2)**	$(-5,0)$ need not be quoted
	N.B. If **(ii)** completed first, subst $y = 0$ into their cartesian eqn (M1) & find x (no f.t.) (A1)		
(ii)	Attempt to eliminate t	M1	
	Simplify to $x = y^2 - 5$ ISW	A1 **(2)**	
(iii)	Attempt to find $\frac{dy}{dx}$ or $\frac{dx}{dy}$ from cartes or para form	M1	Award anywhere in Que
	Obtain $\frac{dy}{dx} = \frac{1}{2t-6}$ or $\frac{1}{2y}$ or $(-)\frac{1}{2}(x+5)^{-\frac{1}{2}}$	A1	
	If $t = 2$, $x = -4$ and $y = -1$	B1	Awarded anywhere in **(iii)**
	Using their num (x, y) & their num $\frac{dy}{dx}$, find tgt eqn	M1	
	$x + 2y + 6 = 0$ AEF(without fractions) ISW	A1 **(5)**	
		9	

5.

(i)	Attempt to connect $dx, d\theta$	M1		But not $dx = d\theta$
	$dx = 2\sin\theta\cos\theta\, d\theta$	A1		AEF
	$\sqrt{\frac{x}{1-x}} = \frac{\sin\theta}{\cos\theta}$	B1		Ignore any references to $\pm$.
	Reduction to $\int 2\sin^2\theta\, d\theta$	A1	**4**	**AG** WWW
(ii)	$\sin^2\theta = k(+/-1+/-\cos 2\theta)$	M1		Attempt to change $(2)\sin^2\theta$ into $f(\cos 2\theta)$
	$2\sin^2\theta = 1 - \cos 2\theta$	A1		Correct attempt
	$\int \cos 2\theta\, d\theta = \frac{1}{2}\sin 2\theta$	B1		Seen anywhere in this part
	Attempting to change limits	M1		<u>Or</u> Attempting to resubstitute; Accept degrees
	$\frac{1}{2}\pi$	A1	**5**	
	<u>Alternatively</u> Parts once & use			
	$\cos^2\theta = 1 - \sin^2\theta$	(M2)		Instead of the M1 A1 B1
	$\frac{1}{2}(\theta - \sin\theta\cos\theta)$	(A1)		Then the final M1 A1 for use of limits

6.

(i) Sep variables eg $\int \frac{1}{6-h}(dh) = \int \frac{1}{20}(dt)$	*M1		s.o.i. Or $\frac{dt}{dh} = \frac{20}{6-h}$ $\rightarrow$M1
LHS = $-\ln(6-h)$	A1		& then $t = -20\ln(6-h)$ (+c)$\rightarrow$ A1+A1
RHS = $\frac{1}{20}t$ (+c)	A1		
Subst $t = 0, h = 1$ into equation containing 'c'	dep*M1		
Correct value of their c = $-(20)\ln 5$ **WWW**	A1		or $(20)\ln 5$ if on LHS
Produce $t = 20\ln\frac{5}{6-h}$ **WWW AG**	A1	**6**	Must see $\ln 5 - \ln(6-h)$
(ii) When $h = 2$, $t = 20\ln\frac{5}{4}$ =4.46(2871)	B1	**1**	Accept 4.5, $4\frac{1}{2}$
(iii) Solve $10 = 20\ln\frac{5}{6-h}$ to $\frac{5}{6-h} = e^{0.5}$	M1		or $\frac{6-h}{5} = e^{-0.5}$ or suitable $\frac{1}{2}$-way stage
h = 2.97(2.9673467…)	A1	**2**	$6 - 5e^{-0.5}$ or $6 - e^{1.109}$
[In (ii),(iii) accept non-decimal (exact) answers but -1 once.**]** Accept truncated values in (ii),(iii).			
(iv) Any indication of (approximately) 6 (m)	B1	**1**	
			10