

## Topic X7 Further calculus (Post-TT B) [44]

1.

Given that

$$\int_0^{\ln 4} (ke^{3x} + (k-2)e^{-\frac{1}{2}x}) dx = 185,$$

find the value of the constant  $k$ .

[7]

(Total 7 marks)

2.

The parametric equations of a curve are

$$x = 2 + 3 \sin \theta \quad \text{and} \quad y = 1 - 2 \cos \theta \quad \text{for} \quad 0 \leq \theta \leq \frac{1}{2}\pi.$$

(i) Find the coordinates of the point on the curve where the gradient is  $\frac{1}{2}$ .

[5]

(ii) Find the cartesian equation of the curve.

[2]

(Total 7 marks)

3.

(i) Show that the substitution  $u = \sqrt{x}$  transforms  $\int \frac{1}{x(1+\sqrt{x})} dx$  to  $\int \frac{2}{u(1+u)} du$ .

[3]

(ii) Hence find the exact value of  $\int_1^9 \frac{1}{x(1+\sqrt{x})} dx$ .

[5]

(Total 8 marks)

4.

Earth is being added to a pile so that, when the height of the pile is  $h$  metres, its volume is  $V$  cubic metres, where

$$V = (h^6 + 16)^{\frac{1}{2}} - 4.$$

(i) Find the value of  $\frac{dV}{dh}$  when  $h = 2$ .

[3]

(ii) The volume of the pile is increasing at a constant rate of 8 cubic metres per hour. Find the rate, in metres per hour, at which the height of the pile is increasing at the instant when  $h = 2$ . Give your answer correct to 2 significant figures.

[3]

(Total 6 marks)

5.

Find  $\int x \sec^2 x dx$ .

[4]

(Total 4 marks)

6.

A bacterial culture has area  $p \text{ mm}^2$  at time  $t$  hours after the culture was placed onto a circular dish.

A scientist states that at time  $t$  hours, the rate of increase of the area of the culture can be modelled as being proportional to the area of the culture.

(a) Show that the scientist's model for  $p$  leads to the equation

$$p = ae^{kt}$$

where  $a$  and  $k$  are constants.

(4)

The scientist measures the values for  $p$  at regular intervals during the first 24 hours after the culture was placed onto the dish.

She plots a graph of  $\ln p$  against  $t$  and finds that the points on the graph lie close to a straight line with gradient 0.14 and vertical intercept 3.95

(b) Estimate, to 2 significant figures, the value of  $a$  and the value of  $k$ .

(3)

(c) Hence show that the model for  $p$  can be rewritten as

$$p = ab^t$$

stating, to 3 significant figures, the value of the constant  $b$ .

(2)

With reference to this model,

(d) (i) interpret the value of the constant  $a$ ,

(ii) interpret the value of the constant  $b$ .

(2)

(e) State a long term limitation of the model for  $p$ .

(1)

(Total 12 marks)