Topic X7 Further calculus (Post-TT B) [44] MARKSCHEME

1.

Integrate e^{3x} to obtain $\frac{1}{3}e^{3x}$ or $e^{-\frac{1}{2}x}$ to obtain $-2e^{-\frac{1}{2}x}$	B1	or both
Obtain indefinite integral of form $m_1e^{3x}+m_2e^{-\frac{1}{2}x}$	M1	any constants m_1 and m_2
Obtain correct $\frac{1}{3}ke^{3x}-2(k-2)e^{-\frac{1}{2}x}$	A1	or equiv
Obtain $e^{3\ln 4}=64$ or $e^{-\frac{1}{2}\ln 4}=\frac{1}{2}$	B1	or both
Apply limits and equate to 185	M1	including substitution of lower limit
Obtain $\frac{64}{3}k-(k-2)-\frac{1}{3}k+2(k-2)=185$	A1	or equiv
Obtain $\frac{17}{2}$	A1	7 or equiv

7

2.

(i)		their $\frac{dy}{d\theta}\Big/\frac{dx}{d\theta}$ $\frac{dy}{dx}=\frac{2\sin\theta}{3\cos\theta}$ their $\frac{dy}{dx}=\frac{1}{2}$ $\tan\theta=\frac{3}{4}$ $(3.8,-0.6)$ or $\left(\frac{19}{5},-\frac{3}{5}\right)$ or $x=3.8, y=-0.6$	M1 A1 M1 A1 A1 [5]	If $\tan\theta=\frac{3}{4}$ not seen, award this A1 only if coords are correct	
(ii)		Manipulating equations into form $\sin\theta=f(x)$ and $\cos\theta=g(y)$ and then using $\sin^2\theta+\cos^2\theta=1$ $\frac{(x-2)^2}{9}+\frac{(1-y)^2}{4}=1$ oe www ISW Accept e.g. $\left(\frac{x-2}{3}\right)^2$ $4x^2+9y^2-16x-18y-11=0$	M1 A1 [2]	If part (ii) is attempted first, and then part (i), allow B1 for obtaining $\frac{dy}{dx}=\frac{4(x-2)}{9(y-1)}$ M1 for equating their $\frac{dy}{dx}$ to $\frac{1}{2}$ A1 for obtaining $9y-8x=-7$ M1 for eliminating x or y from above eqn... A1 for $(3.8,-0.6)$	the following marks in part (i):- ....and their Cartesian equation

3.

(i)	Attempt to connect du and dx, find $\frac{du}{dx}$ or $\frac{dx}{du}$	M1	But not e.g. $du = dx$
	Any correct relationship, however used, such as $dx = 2u\,du$	A1	or $\frac{du}{dx} = \frac{1}{2}x^{-½}$
	Subst with clear reduction (≥ 1 inter step) to **AG**	A1 (3)	WWW

(ii)	Attempt partial fractions	M1	
	$\frac{2}{u} - \frac{2}{1+u}$	A1	
	$\sqrt{}\ A \ln u + B \ln(1+u)$	√A1	Based on $\frac{A}{u} + \frac{B}{1+u}$
	Attempt integ, change limits & use on f(u)	M1	or re-subst & use 1 & 9
	$\ln\frac{9}{4}$ AEexactF (e.g. 2 ln 3 –2 ln 4 + 2 ln 2)	A1 (5)	Not involving ln 1
		8	

4.

(i)	Obtain derivative of form $kh^5(h^6+16)^n$	**M1**	any constant k; any $n < \frac{1}{2}$; allow if – 4 term retained
	Obtain correct $3h^5(h^6+16)^{-\frac{1}{2}}$	**A1**	or (unsimplified) equiv; no –4 now
	Substitute to obtain 10.7	**A1 3**	or greater accuracy or exact equiv
(ii)	Attempt multn or divn using 8 and answer from (i)	**M1**	
	Attempt 8 divided by answer from (i)	**M1**	
	Obtain 0.75	**A1√ 3**	or greater accuracy; allow 0.75 ± 0.01; following their answer from (i)

5.

Use parts with $u = x, dv = \sec^2 x$	M1	result $f(x) + / - \int g(x)\,dx$	
Obtain correct result $x\tan x - \int \tan x\,dx$	A1		
$\int \tan x\,dx = k\ln\sec x$ or $k\ln\cos x$, where $k = 1$ or -1	B1	or $k\ln\lvert\sec x\rvert$ or $k\ln\lvert\cos x\rvert$	
Final answer $= x\tan x - \ln\lvert\sec x\rvert + c$ or $x\tan x + \ln\lvert\cos x\rvert + c$	A1		
	4		

6.

Question	Scheme	Marks	AOs
7(a)	$\frac{\mathrm{d}p}{\mathrm{d}t} \propto p \Rightarrow \frac{\mathrm{d}p}{\mathrm{d}t} = kp$	B1	3.3
	$\int \frac{1}{p}\mathrm{d}p = \int k\,\mathrm{d}t$	M1	1.1b
	$\ln p = kt \{+ c\}$	A1	1.1b
	$\ln p = kt + c \Rightarrow p = \mathrm{e}^{kt+c} = \mathrm{e}^{kt}\mathrm{e}^{c} \Rightarrow p = a\mathrm{e}^{kt}$ *	A1 *	2.1
		(4)	
(b)	$p = a\mathrm{e}^{kt} \Rightarrow \ln p = \ln a + kt$ and evidence of understanding that either • gradient $= k$ or $"M" = k$ • vertical intercept $= \ln a$ or $"C" = \ln a$	M1	2.1
	gradient $= k = 0.14$	A1	1.1b
	vertical intercept $= \ln a = 3.95 \Rightarrow a = \mathrm{e}^{3.95} = 51.935 = 52\ (2\text{ sf})$	A1	1.1b
		(3)	
(c)	e.g. • $p = a\mathrm{e}^{kt} \Rightarrow p = a(\mathrm{e}^{k})^{t} = ab^{t}$, • $p = 52\mathrm{e}^{0.14t} \Rightarrow p = 52(\mathrm{e}^{0.14})^{t}$	B1	2.2a
	$b = 1.15$ which can be implied by $p = 52(1.15)^{t}$	B1	1.1b
		(2)	
(d)(i)	Initial area (i.e. $"52"\ \mathrm{mm}^2$) of bacterial culture that was first placed onto the circular dish.	B1	3.4
(d)(ii)	E.g. • Rate of increase per hour of the area of bacterial culture • The area of bacterial culture increases by “15%” each hour	B1	3.4
		(2)	
(e)	The model predicts that the area of the bacteria culture will increase indefinitely, but the size of the circular dish will be a constraint on this area.	B1	3.5b
		(1)	
			(12 marks)
			Question 7 Notes: