

Topic X7 Further integration (Pre-TT) [46]

1.

(i) Express $\frac{3-2x}{x(3-x)}$ in partial fractions. [3]

(ii) Show that $\int_1^2 \frac{3-2x}{x(3-x)} dx = 0$. [4]

(iii) What does the result of part (ii) indicate about the graph of $y = \frac{3-2x}{x(3-x)}$ between $x = 1$ and $x = 2$? [1]

(Total 8 marks)

2.

Find the exact value of $\int_0^{\frac{1}{8}\pi} (1 - \sin 3x)^2 dx$. [7]

(Total 7 marks)

3.

(i) Show that the substitution $u = e^x + 1$ transforms $\int \frac{e^{2x}}{e^x + 1} dx$ to $\int \frac{u-1}{u} du$. [3]

(ii) Hence show that $\int_0^1 \frac{e^{2x}}{e^x + 1} dx = e - 1 - \ln\left(\frac{e+1}{2}\right)$. [5]

(Total 8 marks)

4.

Show that

$$\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1 + \cos \theta} d\theta = 2 - 2 \ln 2$$

(7)

(Total 7 marks)

5.

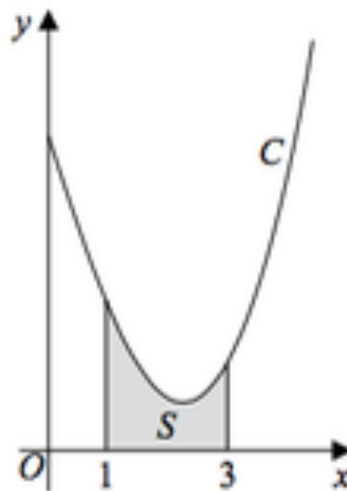


Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 5, \quad x > 0$$

The finite region S , shown shaded in Figure 4, is bounded by the curve C , the line with equation $x = 1$, the x -axis and the line with equation $x = 3$

The table below shows corresponding values of x and y with the values of y given to 4 decimal places as appropriate.

x	1	1.5	2	2.5	3
y	3	2.3041	1.9242	1.9089	2.2958

- (a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of S , giving your answer to 3 decimal places. (3)
- (b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S . (1)
- (c) Show that the exact area of S can be written in the form $\frac{a}{b} + \ln c$, where a , b and c are integers to be found. (6)

(In part c, solutions based entirely on graphical or numerical methods are not acceptable.)

(Total 10 marks)

6.

Find the exact value of $\int_0^1 x^2 e^x dx$.

[6]

(Total 6 marks)