Topic X7 Further integration (Pre-TT) [46] MARKSCHEME

1.

(i)	$\frac{A}{x}+\frac{B}{3-x}$ & c-u rule or $A(3-x)+Bx\equiv 3-2x$	M1		Correct format + suitable method
	$\frac{1}{x}$	A1		seen in (i) or (ii)
	$-\frac{1}{3-x}$	A1	3	ditto; $\frac{1}{x}-\frac{1}{3-x}$ scores 3 immediately
(ii)	$\int\frac{1}{x}(\mathrm{d}x)=\ln x$ or $\ln\lvert x\rvert$	B1		
	$\int\frac{1}{3-x}(\mathrm{d}x)=-\ln(3-x)$ or $-\ln\lvert 3-x\rvert$	B1		Check sign carefully; do not allow $\ln(x-3)$
	Correct method idea of substitution of limits	M1		Dep on an attempt at integrating
	$\ln 2\ (+\ln 1\ -\ln 1)\ -\ln 2\ =0$	A1	4	Clearly seen; WWW **AG**
	Alternative Method			
	If ignoring PFs, $\ln x(3-x)$ immediately	B2		$\ln x(x-3)\to 0$
	As before	M1,A1	(4)	
(iii)	Suitable statement or clear implication e.g. Equal amounts (of area) above and below (axis) or graph crosses axis or there's a root (Be lenient)	B1	1	

2.

Satisfactory start method eg attempt square of $(1-\sin 3x)$	M1	Not e.g. $\frac{(1-\sin 3x)^3}{3}$.
[N.B. The squaring process might include a term $\sin^2 9x$]		
The next 2 marks are awarded for integrating $-2\sin 3x$		
Obtain $\int -2\sin 3x\,\mathrm{d}x=\frac{2}{3}\cos 3x$	*A1	
Obtain $-\frac{2}{3}$ or $(\ldots+0\ldots)-(\ldots+\frac{2}{3}\ldots)$	A1dep*	
The next 3 marks are awarded for integrating $\sin^2 3x$		or for integrating $\sin^2 ax$ where $a=6$ or 9 **only**
Use $\sin^2 3x=k(+/-1+/-\cos 6x)$	M1	$\sin^2 ax=k(+/-1+/-\cos 2ax)$
Correct version $=\frac{1}{2}(1-\cos 6x)$	A1	Correct $=\frac{1}{2}(1-\cos 2ax)$
$\int\cos 6x\,\mathrm{d}x=\frac{1}{6}\sin 6x$, seen anywhere, indep	B1	or $\int\cos 2ax\,\mathrm{d}x=\frac{1}{2a}\sin 2ax$
Final answer $=\frac{1}{4}\pi+$ *their* $-\frac{2}{3}$	A1	Check that the $\frac{1}{4}\pi$ is from $\left[\frac{3}{2}x-\frac{1}{12}\sin 6x\right]_0^{\frac{1}{6}\pi}$
	[7]	

3.

(i)	Attempt to connect $\mathrm{d}u$ and $\mathrm{d}x$ e.g. $\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x$	M1		But not $\mathrm{d}u=\mathrm{d}x$
	Use of $\mathrm{e}^{2x}=(\mathrm{e}^x)^2$ or $(u-1)^2$ s.o.i.	A1		
	Simplification to $\int\frac{u-1}{u}(\mathrm{d}u)$ WWW	A1	3	**AG**
(ii)	Change $\frac{u-1}{u}$ to $1-\frac{1}{u}$ or use parts	M1		If parts, may be twice if $\int\ln x\,\mathrm{d}x$ is involved
	$\int\frac{1}{u}\mathrm{d}u=\ln u$	A1		Seen anywhere in this part
	Either attempt to change limits or resubstitute	M1 (indep)		Expect new limits e+1 & 2
	Show as $\mathrm{e}+1-\ln(\mathrm{e}+1)-\{2\text{ or }(1+1)\}+\ln 2$	A1		
	WWW show final result as $\mathrm{e}-1-\ln\left(\frac{\mathrm{e}+1}{2}\right)$	A1	5	**AG**

4.

12	$\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1+\cos\theta}\,d\theta$		
	Attempts this question by applying the substitution $u = 1+\cos\theta$ and progresses as far as achieving $\int \ldots \frac{(u-1)}{u} \ldots$	M1	3.1a
	$u = 1+\cos\theta \Rightarrow \frac{du}{d\theta} = -\sin\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$	M1	1.1b
	$\left\{\int \frac{\sin 2\theta}{1+\cos\theta}\,d\theta = \right\} \int \frac{2\sin\theta\cos\theta}{1+\cos\theta}\,d\theta = \int \frac{-2(u-1)}{u}\,du$	A1	2.1
	$-2\int\left(1-\frac{1}{u}\right)du = -2(u-\ln u)$	M1	1.1b
		M1	1.1b
	$\left\{\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1+\cos\theta}\,d\theta = \right\} = -2\left[u-\ln u\right]_2^1 = -2((1-\ln 1)-(2-\ln 2))$	M1	1.1b
	$= -2(-1+\ln 2) = 2-2\ln 2$ *	A1*	2.1

5.

14(a)	Uses or implies $h = 0.5$	B1	1.1b
	For correct form of the trapezium rule =	M1	1.1b
	$\frac{0.5}{2}\{3+2.2958+2(2.3041+1.9242+1.9089)\} = 4.393$	A1	1.1b
(b)	Any valid statement reason, for example • Increase the number of strips • Decrease the width of the strips • Use more trapezia	B1	2.4
(c)	For integration by parts on $\int x^2 \ln x\,dx$	M1	2.1
	$= \frac{x^3}{3}\ln x - \int \frac{x^2}{3}\,dx$	A1	1.1b
	$\int -2x+5\,dx = -x^2+5x \quad (+c)$	B1	1.1b
	All integration attempted and limits used Area of $S = \int_1^3 \frac{x^2\ln x}{3} - 2x+5\,dx = \left[\frac{x^3}{9}\ln x - \frac{x^3}{27} - x^2 + 5x\right]_{x=1}^{x=3}$	M1	2.1
	Uses correct ln laws, simplifies and writes in required form	M1	2.1
	Area of $S = \frac{28}{27} + \ln 27 \quad (a = 28, b = 27, c = 27)$	A1	1.1b

6.

Use parts with $u = x^2, \mathrm{d}v = \mathrm{e}^x$	*M1	obtaining a result $f(x) +/- \int g(x)(\mathrm{d}x)$
Obtain $x^2\mathrm{e}^x - \int 2x\mathrm{e}^x\ (\mathrm{d}x)$	A1	
Attempt parts again with $u = (-)(2)x, \mathrm{d}v = \mathrm{e}^x$	M1	
Final = $(x^2 - 2x + 2)\mathrm{e}^x$ AEF incl brackets	A1	s.o.i. eg $\mathrm{e} + (-2x + 2)\mathrm{e}^x$
Use limits correctly throughout	dep*M1	Tolerate (their value for $x = 1$) (-0)
$\mathrm{e}^{(1)} - 2$ ISW Exact answer only	A1 **6**	Allow 0.718 → M1
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