Topic X7 Further calculus (Pre-TT A) [43]

Covering the chapters 11-12, up to and including differentiation of parametric equations.

1.

(i) Express $\dfrac{3-2x}{x(3-x)}$ in partial fractions. [3]

(ii) Show that $\displaystyle\int_1^2 \frac{3-2x}{x(3-x)}\,dx = 0$. [4]

(iii) What does the result of part **(ii)** indicate about the graph of $y = \dfrac{3-2x}{x(3-x)}$ between $x = 1$ and $x = 2$? [1]

(Total 8 marks)

2.

Find the exact value of $\displaystyle\int_0^{\frac{1}{6}\pi} (1-\sin 3x)^2\,dx$. [7]

(Total 7 marks)

3.

(i) Show that the substitution $u = e^x + 1$ transforms $\displaystyle\int \frac{e^{2x}}{e^x+1}\,dx$ to $\displaystyle\int \frac{u-1}{u}\,du$. [3]

(ii) Hence show that $\displaystyle\int_0^1 \frac{e^{2x}}{e^x+1}\,dx = e - 1 - \ln\left(\frac{e+1}{2}\right)$. [5]

(Total 8 marks)

4.

A curve is defined by the parametric equations

$$x = \sin^2\theta, \quad y = 4\sin\theta - \sin^3\theta,$$

where $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{4-3\sin^2\theta}{2\sin\theta}$. **[3]**

(ii) Find the coordinates of the point on the curve at which the gradient is 2. **[3]**

(iii) Show that the curve has no stationary points. **[2]**

(iv) Find a cartesian equation of the curve, giving your answer in the form $y^2 = f(x)$. **[2]**

(Total 10 marks)

5.

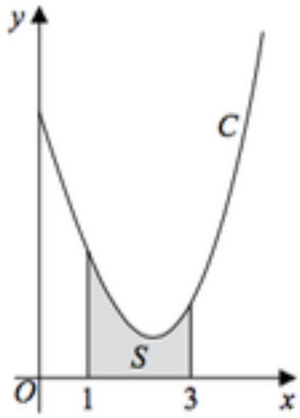

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 5, \quad x > 0$$

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the line with equation $x = 1$, the x-axis and the line with equation $x = 3$

The table below shows corresponding values of x and y with the values of y given to 4 decimal places as appropriate.

x	1	1.5	2	2.5	3
y	3	2.3041	1.9242	1.9089	2.2958

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of S, giving your answer to 3 decimal places.

(3)

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S.

(1)

(c) Show that the exact area of S can be written in the form $\frac{a}{b} + \ln c$, where a, b and c are integers to be found.

(6)

(In part c, solutions based entirely on graphical or numerical methods are not acceptable.)

(Total 10 marks)