

Topic X7 Further calculus (Pre-TT A) [43] MARKSCHEME

1.

<p>(i) $\frac{A}{x} + \frac{B}{3-x}$ & c-u rule or $A(3-x) + Bx \equiv 3 - 2x$</p> <p>$\frac{1}{x}$</p> <p>$-\frac{1}{3-x}$</p>	<p>M1</p> <p>A1</p> <p>A1</p>	<p>Correct format + suitable method</p> <p>seen in (i) or (ii)</p> <p>3 ditto; $\frac{1}{x} - \frac{1}{3-x}$ scores 3 immediately</p>

<p>(ii) $\int \frac{1}{x} (dx) = \ln x$ or $\ln x$</p> <p>$\int \frac{1}{3-x} (dx) = -\ln(3-x)$ or $-\ln 3-x$</p> <p>Correct method idea of substitution of limits $\ln 2 (+ \ln 1 - \ln 1) - \ln 2 = 0$</p> <p><u>Alternative Method</u> If ignoring PFs, $\ln x(3-x)$ immediately As before</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>B2</p> <p>M1,A1 (4)</p>	<p>Check sign carefully; do not allow $\ln(x-3)$</p> <p>Dep on an attempt at integrating</p> <p>4 Clearly seen; WWW AG</p> <p>$\ln x(x-3) \rightarrow 0$</p>

<p>(iii) Suitable statement or clear implication e.g. Equal amounts (of area) above and below (axis) or graph crosses axis or there's a root (Be lenient)</p>	<p>B1</p>	<p>1</p>

2.

<p>Satisfactory start method eg attempt square of $(1 - \sin 3x)$ [N.B. The squaring process might include a term $\sin^2 9x$] <u>The next 2 marks are awarded for integrating $-2\sin 3x$</u> Obtain $\int -2 \sin 3x dx = \frac{2}{3} \cos 3x$ Obtain $-\frac{2}{3}$ or $(\dots + 0 \dots) - (\dots + \frac{2}{3} \dots)$ <u>The next 3 marks are awarded for integrating $\sin^2 3x$</u> Use $\sin^2 3x = k(+/-1 +/- \cos 6x)$ Correct version = $\frac{1}{2}(1 - \cos 6x)$ $\int \cos 6x dx = \frac{1}{6} \sin 6x$, seen anywhere, indep Final answer = $\frac{1}{4} \pi + \text{their} - \frac{2}{3}$</p>	<p>M1</p> <p>*A1</p> <p>A1dep*</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>A1</p> <p>[7]</p>	<p>Not e.g. $\frac{(1 - \sin 3x)^2}{3}$.</p> <p>or for integrating $\sin^2 ax$ where $a = 6$ or 9 only $\sin^2 ax = k(+/-1 +/- \cos 2ax)$ Correct = $\frac{1}{2}(1 - \cos 2ax)$ or $\int \cos 2ax dx = \frac{1}{2a} \sin 2ax$ Check that the $\frac{1}{4} \pi$ is from $\left[\frac{3}{2}x - \frac{1}{12} \sin 6x \right]_0^{\frac{1}{2}\pi}$</p>
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3.

<p>(i) Attempt to connect du and dx e.g. $\frac{du}{dx} = e^x$</p> <p>Use of $e^{2x} = (e^x)^2$ or $(u-1)^2$ s.o.i.</p> <p>Simplification to $\int \frac{u-1}{u} (du)$ WWW</p>	<p>M1</p> <p>A1</p> <p>A1</p>	<p>But not $du = dx$</p> <p>3 AG</p>

<p>(ii) Change $\frac{u-1}{u}$ to $1 - \frac{1}{u}$ or use parts</p> <p>$\int \frac{1}{u} du = \ln u$</p> <p>Either attempt to change limits or resubstitute Show as $e+1 - \ln(e+1) - \{2 \text{ or } (1+1)\} + \ln 2$</p> <p>WWW show final result as $e-1 - \ln\left(\frac{e+1}{2}\right)$</p>	<p>M1</p> <p>A1</p> <p>M1 (indep)</p> <p>A1</p> <p>A1</p>	<p>If parts, may be twice if $\int \ln x dx$ is involved</p> <p>Seen anywhere in this part</p> <p>Expect new limits $e+1$ & 2</p> <p>5 AG</p>

4.

(i)	$\frac{dy}{dx} = \text{attempt at } \frac{dy}{dx} \text{ but not } \frac{4-3\sin^2\theta}{2\sin\theta}$ $\frac{dy}{dx} = \text{attempt at } \frac{dy}{dx}$ $4\cos\theta - 3\sin^2\theta\cos\theta$ seen $\left(\frac{dy}{dx}\right) = \frac{4\cos\theta - 3\sin^2\theta\cos\theta}{2\sin\theta\cos\theta} = \frac{4-3\sin^2\theta}{2\sin\theta}$ AG	M1 B1 A1 [3]	indep	Alternative Change to Cartesian form, differentiate and resubstitute Correct differentiation of correct equation
(ii)	Equating given $\frac{dy}{dx}$ to 2 & producing quadratic equation $\sin\theta = \frac{2}{3}$ P is $\left(\frac{4}{9}, \frac{64}{27}\right)$	M1 A1 A1 [3]	ignore any other given value Accept 0.444... and 2.37... or better	
(iii)	Identify problem as solving $4-3\sin^2\theta=0$ Show convincingly that $4-3\sin^2\theta=0$ has no solutions	M1 A1 [2]	Consider magnitude of $\sin\theta$	
(iv)	Attempt to eliminate $\sin\theta$ from the 2 given equations Produce $y^2 = x(4-x)^2$ or $16x-8x^2+x^3$	M1 A1 [2]	e.g. $y = 4\sqrt{x} - (\sqrt{x})^3$ ISW	

5.

14(a)	Uses or implies $h = 0.5$	B1	1.1b
	For correct form of the trapezium rule =	M1	1.1b
	$\frac{0.5}{2} \{3 + 2.2958 + 2(2.3041 + 1.9242 + 1.9089)\} = 4.393$	A1	1.1b
		(3)	
(b)	Any valid statement reason, for example <ul style="list-style-type: none"> • Increase the number of strips • Decrease the width of the strips • Use more trapezia 	B1	2.4
		(1)	
(c)	For integration by parts on $\int x^2 \ln x \, dx$	M1	2.1
	$= \frac{x^3}{3} \ln x - \int \frac{x^2}{3} \, dx$	A1	1.1b
	$\int -2x + 5 \, dx = -x^2 + 5x \quad (+c)$	B1	1.1b
	All integration attempted and limits used		
	Area of $S = \int_1^3 \frac{x^2 \ln x}{3} - 2x + 5 \, dx = \left[\frac{x^3}{9} \ln x - \frac{x^3}{27} - x^2 + 5x \right]_{x=1}^{x=3}$	M1	2.1
	Uses correct ln laws, simplifies and writes in required form	M1	2.1
Area of $S = \frac{28}{27} + \ln 27 \quad (a = 28, b = 27, c = 27)$	A1	1.1b	
	(6)		
(10 marks)			