

Topic X7 Further calculus (Pre-TT B) [45] MARKSCHEME

1.

Question	Scheme	Marks	AOs
11 (a)	$f'(x) = k - 4x - 3x^2$		
	$f''(x) = -4 - 6x = 0$	M1	1.1b
	<p>Criteria 1</p> <p>Either</p> $f''(x) = -4 - 6x = 0 \Rightarrow x = \frac{4}{-6} \Rightarrow x = -\frac{2}{3}$ <p>or</p> $f''\left(-\frac{2}{3}\right) = -4 - 6\left(-\frac{2}{3}\right) = 0$ <p>Criteria 2</p> <p>Either</p> <ul style="list-style-type: none"> • $f''(-0.7) = -4 - 6(-0.7) = 0.2 > 0$ • $f''(-0.6) = -4 - 6(-0.6) = -0.4 < 0$ <p>or</p> <ul style="list-style-type: none"> • $f'''\left(-\frac{2}{3}\right) = -6 \neq 0$ 		
	At least one of Criteria 1 or Criteria 2	B1	2.4
	Both Criteria 1 and Criteria 2 and concludes C has a point of inflection at $x = -\frac{2}{3}$	A1	2.1
		(3)	
(b)	$f'(x) = k - 4x - 3x^2, AB = 4\sqrt{2}$		
	$f(x) = kx - 2x^2 - x^3 \{+ c\}$	M1	1.1b
		A1	1.1b
	$f(0) = 0$ or $(0, 0) \Rightarrow c = 0 \Rightarrow f(x) = kx - 2x^2 - x^3$ $\{f(x) = 0 \Rightarrow\} f(x) = x(k - 2x - x^2) = 0 \Rightarrow \{x = 0,\} k - 2x - x^2 = 0$	A1	2.2a
	$\{x^2 + 2x - k = 0\} \Rightarrow (x+1)^2 - 1 - k = 0, x = \dots$	M1	2.1
	$\Rightarrow x = -1 \pm \sqrt{k+1}$	A1	1.1b
	$AB = (-1 + \sqrt{k+1}) - (-1 - \sqrt{k+1}) = 4\sqrt{2} \Rightarrow k = \dots$	M1	2.1
	So, $2\sqrt{k+1} = 4\sqrt{2} \Rightarrow k = 7$	A1	1.1b
		(7)	
(10 marks)			

2.

(i)	$\frac{d\theta}{dt} = \dots$	B1	
	$k(160 - \theta)$	B1	(2) The 2 @ 'B1' are indep
(ii)	Separate variables with $(160 - \theta)$ in denom; or invert	*M1	$\int \frac{1}{160 - \theta} d\theta = \int k, \frac{1}{k}, 1 dt$
	Indication that LHS = $\ln f(\theta)$	A1	If wrong ln, final 3@A = 0
	RHS = kt or $\frac{1}{k}t$ or t (+ c)	A1	
	Subst. $t = 0, \theta = 20$ into equation containing 'c'	dep*M1	
	Subst $t = 5, \theta = 65$ into equation containing 'c' & 'k'	dep*M1	
	$c = -\ln 140$ (-4.94)	ISW	A1
	$k = \frac{1}{5} \ln \frac{140}{95}$ (≈ 0.077 or 0.078)	ISW	A1
	Using their 'c' & 'k', subst $t = 10$ & evaluate θ	dep*M1	
	$\theta = 96(95.535714)$ ($95 \frac{15}{28}$)	A1	(9)

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3.

(i)	$\frac{dA}{dt}$ or kA^2 seen	M1	
	$\frac{dA}{dt} = kA^2$	A1	2

(ii)	Separate variables + attempt to integrate	*M1	Accept if based on $\frac{dA}{dt} = kA^2$ or A^2
	$-\frac{1}{A} = kt + c$ or $-\frac{1}{kA} = t + c$ or $-\frac{1}{A} = t + c$	A1	
	Subst one of (0,0), (1,1000) or (2,2000) into eqn.	dep*M1	Equation must contain k and/or c
	Subst another of (0,0), (1,1000) or (2,2000) into eqn	dep*M1	This equation must contain k <u>and</u> c
	Substitute $A = 3000$ into eqn with k and c subst	dep*M1	
	$t = \frac{7}{3}$ ISW	A1	6 Accept 2.33, 2h 20 m

4.

$\int_1^3 \left[(11 - 9x^{-2}) - (x^2 + 1) \right] dx = \left[9x^{-1} - \frac{1}{3}x^3 + 10x \right]_1^3$	M1	Attempt subtraction (correct order) at any point
$= (3 - 9 + 30) - (9 - \frac{1}{3} + 10)$	M1	Attempt integration – inc. in power for at least one term
$= 24 - 18\frac{2}{3}$	A1	Obtain $\pm (-\frac{1}{3}x^3 + 10x)$ or $11x$ and $\frac{1}{3}x^3 + x$
$= 5\frac{1}{3}$	M1	Obtain remaining term of form kx^{-1}
OR	A1	Obtain $\pm 9x^{-1}$ or any unsimplified equiv
$\left[11x + 9x^{-1} \right]_1^3 - \left[\frac{1}{3}x^3 + x \right]_1^3$	M1	Use limits $x = 1, 3$ – correct order & subtraction
$= [(33 + 3) - (11 + 9)] - [(9 + 3) - (\frac{1}{3} + 1)]$	A1	Obtain $5\frac{1}{3}$, or exact equiv
$= 16 - 10\frac{2}{3}$		
$= 5\frac{1}{3}$		

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5.

10 (a)	$x > \ln\left(\frac{4}{3}\right)$	B1	2.2a
(b)	Attempts to apply $\int y \frac{dx}{dt} dt$	M1	3.1a
	$\left\{ \int y \frac{dx}{dt} dt = \right\} = \int \left(\frac{1}{t+1} \right) \left(\frac{1}{t+2} \right) dt$	A1	1.1b
	$\frac{1}{(t+1)(t+2)} \equiv \frac{A}{t+1} + \frac{B}{t+2} \Rightarrow 1 \equiv A(t+2) + B(t+1)$	M1	3.1a
	$\{A = 1, B = -1 \Rightarrow \}$ gives $\frac{1}{t+1} - \frac{1}{t+2}$	A1	1.1b
	$\left\{ \int \left(\frac{1}{t+1} - \frac{1}{t+2} \right) dt = \right\} \ln(t+1) - \ln(t+2)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = [\ln(t+1) - \ln(t+2)]_0^2 = (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$	M1	2.2a
	$= \ln 3 - \ln 4 + \ln 2 = \ln\left(\frac{(3)(2)}{4}\right) = \ln\left(\frac{6}{4}\right)$		
$= \ln\left(\frac{3}{2}\right) *$	A1*	2.1	