Topic X7 Further calculus (Pre-TT C) [45] MARKSCHEME

1.

Use parts with $u = x^2, dv = e^x$	*M1	obtaining a result $f(x)+/-\int g(x)(dx)$
Obtain $x^2e^x - \int 2xe^x\,(dx)$	A1	
Attempt parts again with $u = (-)(2)x, dv = e^x$	M1	
Final = $(x^2-2x+2)e^x$ AEF incl brackets	A1	s.o.i. eg $e+(-2x+2)e^x$
Use limits correctly throughout	dep*M1	Tolerate (their value for $x=1$) (-0)
$e^{(1)}-2$ ISW Exact answer only	A1 **6**	Allow 0.718 → M1
		6

2.

Question	Scheme	Marks	AOs
7	$\left\{\int xe^{2x}\,dx\right\}$, $\left\{\begin{matrix} u = x & \Rightarrow \frac{du}{dx}=1 \\ \frac{dv}{dx}=e^{2x} & \Rightarrow v=\frac{1}{2}e^{2x}\end{matrix}\right\}$		
	$\left\{\int xe^{2x}\,dx\right\} = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\{dx\}$	M1	3.1a
	$\left\{\int 2e^{2x} - xe^{2x}\,dx\right\} = e^{2x} - \left(\frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\{dx\}\right)$	M1	1.1b
	$= e^{2x} - \left(\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right)$	A1	1.1b
	$\text{Area}(R) = \int_0^2 2e^{2x} - xe^{2x}\,dx = \left[\frac{5}{4}e^{2x} - \frac{1}{2}xe^{2x}\right]_0^2$	M1	2.2a
	$= \left(\frac{5}{4}e^4 - e^4\right) - \left(\frac{5}{4}e^{2(0)} - \frac{1}{2}(0)e^0\right) = \frac{1}{4}e^4 - \frac{5}{4}$	A1	2.1
		(5)	

3.

(a)	Obtain integral of form $k(4x-1)^{-1}$	**M1**	any non-zero constant k
	Obtain $-\frac{1}{2}(4x-1)^{-1}$	**A1**	or equiv; allow + c
	Substitute limits and attempt evaluation	**M1**	for any expression of form $k'(4x-1)^n$
	Obtain $\frac{2}{21}$	**A1 4**	or exact equiv
(b)	Integrate to obtain $\ln x$	**B1**	
	Substitute limits to obtain $\ln 2a - \ln a$	**B1**	
	Subtract integral attempt from attempt at area of appropriate rectangle	**M1**	or equiv
	Obtain $1-(\ln 2a - \ln a)$	**A1**	or equiv
	Show at least one relevant logarithm property	**M1**	at any stage of solution
	Obtain $1-\ln 2$ and hence $\ln(\frac{1}{2}e)$	**A1 6**	**AG**; full detail required

4.

Question	Scheme	Marks	AOs
8(a)	$\frac{dV}{dt} = 160\pi,\ V = \frac{1}{3}\pi h^2(75-h) = 25\pi h^2 - \frac{1}{3}\pi h^3$		
	$\frac{dV}{dh} = 50\pi h - \pi h^2$	M1	1.1b
		A1	1.1b
	$\left\{\frac{dV}{dh}\times\frac{dh}{dt} = \frac{dV}{dt} \Rightarrow\right\}\ (50\pi h - \pi h^2)\frac{dh}{dt} = 160\pi$	M1	3.1a
	When $h = 10,\ \left\{\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} \Rightarrow\right\}\ \frac{160\pi}{50\pi(10)-\pi(10)^2}\ \left\{=\frac{160\pi}{400\pi}\right\}$	dM1	3.4
	$\frac{dh}{dt} = 0.4\ (\text{cm s}^{-1})$	A1	1.1b
		(5)	
(b)	$\frac{dh}{dt} = \frac{300\pi}{50\pi(20)-\pi(20)^2}$	M1	3.4
	$\frac{dh}{dt} = 0.5\ (\text{cm s}^{-1})$	A1	1.1b
		(2)	

5.

Question	Scheme	Marks	AOs
12	$\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1+\cos\theta}\, d\theta$		
	Attempts this question by applying the substitution $u = 1+\cos\theta$ and progresses as far as achieving $\int \ldots \frac{(u-1)}{u} \ldots$	M1	3.1a
	$u = 1+\cos\theta \Rightarrow \frac{du}{d\theta} = -\sin\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$	M1	1.1b
	$\left\{\int \frac{\sin 2\theta}{1+\cos\theta}\, d\theta = \right\} \int \frac{2\sin\theta\cos\theta}{1+\cos\theta}\, d\theta = \int \frac{-2(u-1)}{u}\, du$	A1	2.1
	$-2\int\left(1-\frac{1}{u}\right) du = -2(u - \ln u)$	M1	1.1b
		M1	1.1b
	$\left\{\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1+\cos\theta}\, d\theta = \right\} = -2\left[u - \ln u\right]_2^1 = -2((1-\ln 1)-(2-\ln 2))$	M1	1.1b
	$= -2(-1+\ln 2) = 2 - 2\ln 2$ *	A1*	2.1
		(7)	

6.

(i)	Attempt to sep variables in the form $\int \frac{p}{(x-8)^{1/3}}\,\mathrm{d}x = \int q\,\mathrm{d}t$	M1		Or invert as $\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{r}{(x-8)^{1/3}}$; p,q,r consts
	$\int \frac{1}{(x-8)^{1/3}}\,\mathrm{d}x = k(x-8)^{2/3}$	A1		k const
	All correct (+ c)	A1		
	For equation containing 'c'; substitute $t = 0$, $x = 72$	M1		M2 for $\int_{72}^{35} = \int_0^t$ or $\int_{35}^{72} = \int_0^t$
	Correct corresponding value of c from correct eqn	A1		
	Subst their c & $x = 35$ back into eqn	M1		
	$t = \frac{21}{8}$ or 2.63 / 2.625 [C.A.O]	A1	**7**	A2: $t = \frac{21}{8}$ or 2.63 / 2.625 WWW
(ii)	State/imply in some way that $x = 8$ when flow stops	B1		
	Substitute $x = 8$ back into equation containing numeric 'c'	M1		
	$t = 6$	A1	**3**	
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