Topic X8 Further calculus (Post-TT) [59]

1.

A curve has equation $y = x^2 + kx - 4x^{-1}$ where k is a constant. Given that the curve has a minimum point when $x = -2$

- find the value of k,
- show that the curve has a point of inflection which is not a stationary point.

[7]

(Total 7 marks)

2.

The parametric equations of a curve are

$$x = 2 + 3\sin\theta \quad \text{and} \quad y = 1 - 2\cos\theta \quad \text{for} \quad 0 \leqslant \theta \leqslant \tfrac{1}{2}\pi.$$

(i) Find the coordinates of the point on the curve where the gradient is $\frac{1}{2}$. [5]

(ii) Find the cartesian equation of the curve. [2]

(Total 7 marks)

3.

Earth is being added to a pile so that, when the height of the pile is h metres, its volume is V cubic metres, where

$$V = (h^6 + 16)^{\frac{1}{2}} - 4.$$

(i) Find the value of $\frac{\mathrm{d}V}{\mathrm{d}h}$ when $h = 2$. [3]

(ii) The volume of the pile is increasing at a constant rate of 8 cubic metres per hour. Find the rate, in metres per hour, at which the height of the pile is increasing at the instant when $h = 2$. Give your answer correct to 2 significant figures. [3]

(Total 6 marks)

4.

A curve has parametric equations

$$x = t^2 - 6t + 4, \quad y = t - 3.$$

Find

(i) the coordinates of the point where the curve meets the x-axis, **[2]**

(ii) the equation of the curve in cartesian form, giving your answer in a simple form without brackets, **[2]**

(iii) the equation of the tangent to the curve at the point where $t = 2$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **[5]**

(Total 9 marks)

5.

The height, h metres, of a shrub t years after planting is given by the differential equation

$$\frac{dh}{dt} = \frac{6-h}{20}.$$

A shrub is planted when its height is 1 m.

(i) Show by integration that $t = 20\ln\left(\frac{5}{6-h}\right)$. [6]

(ii) How long after planting will the shrub reach a height of 2 m? [1]

(iii) Find the height of the shrub 10 years after planting. [2]

(iv) State the maximum possible height of the shrub. [1]

(Total 10 marks)

6.

A bacterial culture has area p mm^2 at time t hours after the culture was placed onto a circular dish.

A scientist states that at time t hours, the rate of increase of the area of the culture can be modelled as being proportional to the area of the culture.

(a) Show that the scientist's model for p leads to the equation

$$p = ae^{kt}$$

where a and k are constants.

(4)

The scientist measures the values for p at regular intervals during the first 24 hours after the culture was placed onto the dish.

She plots a graph of $\ln p$ against t and finds that the points on the graph lie close to a straight line with gradient 0.14 and vertical intercept 3.95

(b) Estimate, to 2 significant figures, the value of a and the value of k.

(3)

(c) Hence show that the model for p can be rewritten as

$$p = ab^t$$

stating, to 3 significant figures, the value of the constant b.

(2)

With reference to this model,

(d) (i) interpret the value of the constant a,

(ii) interpret the value of the constant b.

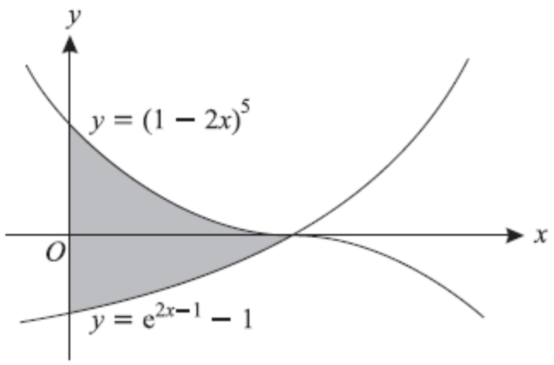
(2)

(e) State a long term limitation of the model for p.

(1)

(Total 12 marks)

7.



The diagram shows the curves $y = (1 - 2x)^5$ and $y = e^{2x-1} - 1$. The curves meet at the point $\left(\frac{1}{2}, 0\right)$. Find the exact area of the region (shaded in the diagram) bounded by the y-axis and by part of each curve. [8]

(Total 8 marks)