Topic X8 Further calculus (Post-TT) [59] MARKSCHEME

1.

$\frac{dy}{dx} = 2x + k + 4x^{-2}$	M1	1.1a	Attempt to differentiate	Power decreases by 1 for at least 2 terms
$2(-2) + k + 4(-2)^{-2} = 0$	M1	3.1a	Substitute $x = -2$, equate to 0 and attempt to solve	
$k = 3$	A1	1.1		
$\frac{d^2y}{dx^2} = 2 - 8x^{-3}$				
$2 - 8x^{-3} = 0$	M1	3.1a	Equate second derivative to 0 and attempt to solve	
$x = 4^{\frac{1}{3}}$	A1	1.1		
for $x < 4^{\frac{1}{3}} \Rightarrow \frac{d^2y}{dx^2} < 0$ for $x > 4^{\frac{1}{3}} \Rightarrow \frac{d^2y}{dx^2} > 0$	E1	2.1	Consider convex/concave either side of $x = 4^{\frac{1}{3}}$ and conclude	
When $x = 4^{\frac{1}{3}}, \frac{dy}{dx} \neq 0$ hence not a stationary point	E1	2.1	Consider gradient at $x = 4^{\frac{1}{3}}$, or justify that $x = -2$ is the only stationary point	
	[7]			

2.

(i)	their $\frac{dy}{d\theta} \Big/ \frac{dx}{d\theta}$	M1		
	$\frac{dy}{dx} = \frac{2\sin\theta}{3\cos\theta}$	A1		
	their $\frac{dy}{dx} = \frac{1}{2}$	M1		
	$\tan\theta = \frac{3}{4}$	A1	If $\tan\theta = \frac{3}{4}$ not seen, award this A1 only if coords are correct	
	$(3.8, -0.6)$ or $\left(\frac{19}{5}, -\frac{3}{5}\right)$ or $x = 3.8, y = -0.6$	A1		
		[5]		
(ii)	Manipulating equations into form $\sin\theta = f(x)$ and $\cos\theta = g(y)$ and then using $\sin^2\theta + \cos^2\theta = 1$	M1	If part (ii) is attempted first, and then part (i), allow B1 for obtaining $\frac{dy}{dx} = \frac{4(x-2)}{9(y-1)}$ M1 for equating their $\frac{dy}{dx}$ to $\frac{1}{2}$	the following marks in part (i):-
	$\frac{(x-2)^2}{9} + \frac{(1-y)^2}{4} = 1$ oe www ISW Accept e.g. $\left(\frac{x-2}{3}\right)^2$ $4x^2 + 9y^2 - 16x - 18y - 11 = 0$	A1	A1 for obtaining $9y - 8x = -7$ M1 for eliminating x or y from above eqn... A1 for $(3.8, -0.6)$)	....and their Cartesian equation
		[2]		

3.

(i)	Obtain derivative of form $kh^5(h^6 + 16)^n$	M1	any constant k, any $n < \frac{1}{2}$; allow if -4 term retained
	Obtain correct $3h^5(h^6 + 16)^{-\frac{1}{2}}$	A1	or (unsimplified) equiv; no -4 now
	Substitute to obtain 10.7	A1 3	or greater accuracy or exact equiv
(ii)	Attempt multn or divn using 8 and answer from (i)	M1	
	Attempt 8 divided by answer from (i)	M1	
	Obtain 0.75	A1√ 3	or greater accuracy; allow 0.75 ± 0.01; following their answer from (i)

4.

(i)	Solve $0 = t-3$ & subst into $x = t^2 - 6t + 4$	M1	
	Obtain $x = -5$	A1 **(2)**	$(-5,0)$ need not be quoted
	N.B. If **(ii)** completed first, subst $y = 0$ into their cartesian eqn (M1) & find x (no f.t.) (A1)		

(ii)	Attempt to eliminate t	M1	
	Simplify to $x = y^2 - 5$ ISW	A1 **(2)**	

(iii)	Attempt to find $\frac{dy}{dx}$ or $\frac{dx}{dy}$ from cartes or para form	M1	Award anywhere in Que
	Obtain $\frac{dy}{dx} = \frac{1}{2t-6}$ or $\frac{1}{2y}$ or $(-)\frac{1}{2}(x+5)^{-\frac{1}{2}}$	A1	
	If $t = 2$, $x = -4$ and $y = -1$	B1	Awarded anywhere in **(iii)**
	Using their num (x, y) & their num $\frac{dy}{dx}$, find tgt eqn	M1	
	$x + 2y + 6 = 0$ AEF(without fractions) ISW	A1 **(5)**	
		9	

5.

(i) Sep variables eg $\int \frac{1}{6-h}(dh) = \int \frac{1}{20}(dt)$	*M1		s.o.i. Or $\frac{dt}{dh} = \frac{20}{6-h} \to$ M1
LHS = $-\ln(6-h)$	A1		& then $t = -20\ln(6-h)$ (+c) $\to$ A1+A1
RHS = $\frac{1}{20}t$ (+c)	A1		
Subst $t = 0, h = 1$ into equation containing 'c'	dep*M1		
Correct value of their c = $-(20)\ln 5$ **WWW**	A1		or $(20)\ln 5$ if on LHS
Produce $t = 20\ln\frac{5}{6-h}$ **WWW AG**	A1	**6**	Must see $\ln 5 - \ln(6-h)$
(ii) When $h = 2$, $t = 20\ln\frac{5}{4}$ =4.46(2871)	B1	**1**	Accept 4.5, $4\frac{1}{2}$
(iii) Solve $10 = 20\ln\frac{5}{6-h}$ to $\frac{5}{6-h} = e^{0.5}$	M1		or $\frac{6-h}{5} = e^{-0.5}$ or suitable $\frac{1}{2}$-way stage
h = 2.97(2.9673467…)	A1	**2**	$6 - 5e^{-0.5}$ or $6 - e^{1.109}$
[In (ii),(iii) accept non-decimal (exact) answers but -1 once.**]** Accept truncated values in (ii),(iii).			
(iv) Any indication of (approximately) 6 (m)	B1	**1**	
			10

6.

7(a)	$\frac{dp}{dt} \propto p \Rightarrow \frac{dp}{dt} = kp$	B1	3.3
	$\int \frac{1}{p} dp = \int k\, dt$	M1	1.1b
	$\ln p = kt\ \{+c\}$	A1	1.1b
	$\ln p = kt + c \Rightarrow p = e^{kt+c} = e^{kt}e^{c} \Rightarrow p = ae^{kt}$ *	A1 *	2.1
(b)	$p = ae^{kt} \Rightarrow \ln p = \ln a + kt$ and evidence of understanding that either • gradient $= k$ or $"M" = k$ • vertical intercept $= \ln a$ or $"C" = \ln a$	M1	2.1
	gradient $= k = 0.14$	A1	1.1b
	vertical intercept $= \ln a = 3.95 \Rightarrow a = e^{3.95} = 51.935 = 52$ (2 sf)	A1	1.1b
(c)	e.g. • $p = ae^{kt} \Rightarrow p = a(e^{k})^{t} = ab^{t}$, • $p = 52e^{0.14t} \Rightarrow p = 52(e^{0.14})^{t}$	B1	2.2a
	$b = 1.15$ which can be implied by $p = 52(1.15)^{t}$	B1	1.1b
(d)(i)	Initial area (i.e. "52" mm^2) of bacterial culture that was first placed onto the circular dish.	B1	3.4
(d)(ii)	E.g. • Rate of increase per hour of the area of bacterial culture • The area of bacterial culture increases by "15%" each hour	B1	3.4
(e)	The model predicts that the area of the bacteria culture will increase indefinitely, but the size of the circular dish will be a constraint on this area.	B1	3.5b

7.

Obtain integral of form $k(1-2x)^6$	M1		[any non-zero constant k]
Obtain correct $-\frac{1}{12}(1-2x)^6$	A1		[or unsimplified equiv; allow $+ c$]
Use limits to obtain $\frac{1}{12}$	A1		[or exact (unsimplified) equiv]
Obtain integral of form $k e^{2x-1}$	M1		[or equiv; any non-zero constant k]
Obtain correct $\frac{1}{2}e^{2x-1} - x$	A1		[or equiv; allow $+ c$]
Use limits to obtain $-\frac{1}{2}e^{-1}$	A1		[or exact (unsimplified) equiv]
Show correct process for finding required area	M1		[at any stage of solution; if process involves two definite integrals, second must be negative]
Obtain $\frac{1}{12} + \frac{1}{2}e^{-1}$	A1	8	[or exact equiv; no $+ c$]