Topic X8 Mechanics (Post-TT A) [56]

1.

A particle, P, moves with constant acceleration $(\mathbf{i} - 2\mathbf{j})$ m s^{-2}.
At time t = 0 seconds, the particle is at the point A with position vector $(2\mathbf{i} + 5\mathbf{j})$ m and is moving with velocity $\mathbf{u}$ m s^{-1}.

At time t = 3 seconds, P is at the point B with position vector $(-2.5\mathbf{i} + 8\mathbf{j})$ m.

Find $\mathbf{u}$.

(4)

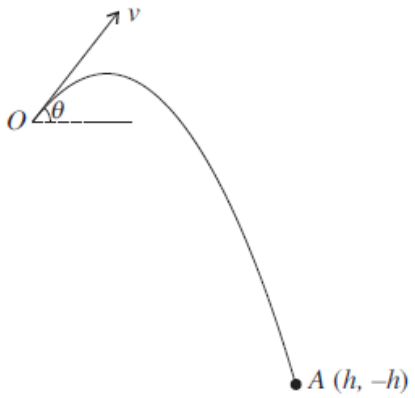
(Total 4 marks)

2.

A particle is projected from a point O with speed v m s^{-1} at an angle of elevation θ above the horizontal and it moves freely under gravity. The horizontal and upward vertical displacements of the particle from O at any subsequent time, t seconds, are x m and y m respectively.

(i) Express x and y in terms of θ and t, and hence show that

$$y = x\tan\theta - \frac{4.9x^2}{v^2\cos^2\theta}.$$ [4]



The particle subsequently passes through the point A with coordinates $(h, -h)$ as shown in the diagram. It is given that $v = 14$ and $\theta = 30°$.

(ii) Calculate h. [4]

(iii) Calculate the direction of motion of the particle at A. [5]

(iv) Calculate the speed of the particle at A. [2]

(Total 15 marks)

3.

A particle, P, moves under the action of a single force in such a way that at time t seconds, where $t \geq 0$, its velocity $\mathbf{v}$ m s^{-1} is given by

$$\mathbf{v} = (t^2 - 3t)\,\mathbf{i} - 12t\,\mathbf{j}$$

The mass of P is 0.5 kg.

Find the time at which the magnitude of the force acting on P is 6.5 N.

(7)

(Total 7 marks)

4.

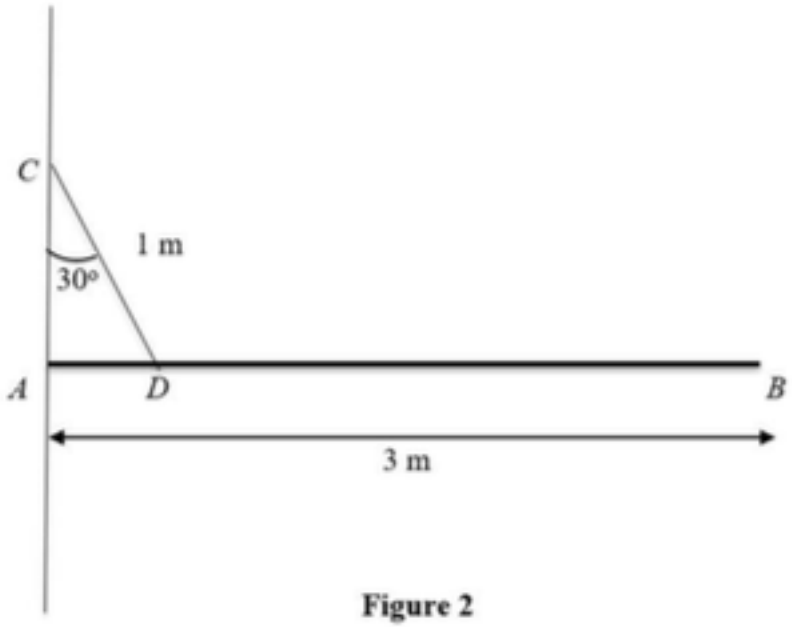


Figure 2

A beam AB, of mass 20 kg and length 3 m, is smoothly hinged to a vertical wall at one end A.

The beam is held in equilibrium in a horizontal position by a rope of length 1 m.
One end of the rope is fixed to a point, C, on the wall which is vertically above A.
The other end of the rope is fixed to the point D on the beam so that angle ACD is 30°, as shown in Figure 2.

The beam is modelled as a uniform rod and the rope is modelled as a light inextensible string.

Using the model, find

(a) the tension in the rope,

(4)

(b) the direction of the force exerted by the wall on the beam at A.

(6)

(c) If the rope were not modelled as being light, state how this would affect the tension in the rope, explaining your answer carefully.

(2)

The rope is now removed and replaced by a longer rope which is still attached to the wall at C but is now attached to the beam at G, where G is the midpoint of AB. The beam AB remains in equilibrium in a horizontal position.

(d) Show that the force exerted by the wall on the beam at A now acts horizontally

(2)

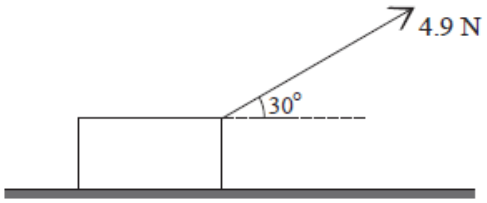
(Total 14 marks)

5.

A block of weight 14.7 N is at rest on a horizontal floor. A force of magnitude 4.9 N is applied to the block.

(i) The block is in limiting equilibrium when the 4.9 N force is applied horizontally. Show that the coefficient of friction is $\frac{1}{3}$. [2]

(ii)



When the force of 4.9 N is applied at an angle of 30° above the horizontal, as shown in the diagram, the block moves across the floor. Calculate

(a) the vertical component of the contact force between the floor and the block, and the magnitude of the frictional force, [5]

(b) the acceleration of the block. [5]

(iii) Calculate the magnitude of the frictional force acting on the block when the 4.9 N force acts at an angle of 30° to the upward vertical, justifying your answer fully. [4]

(Total 16 marks)