Topic X8 Further calculus (Pre-TT A) [45]

1.

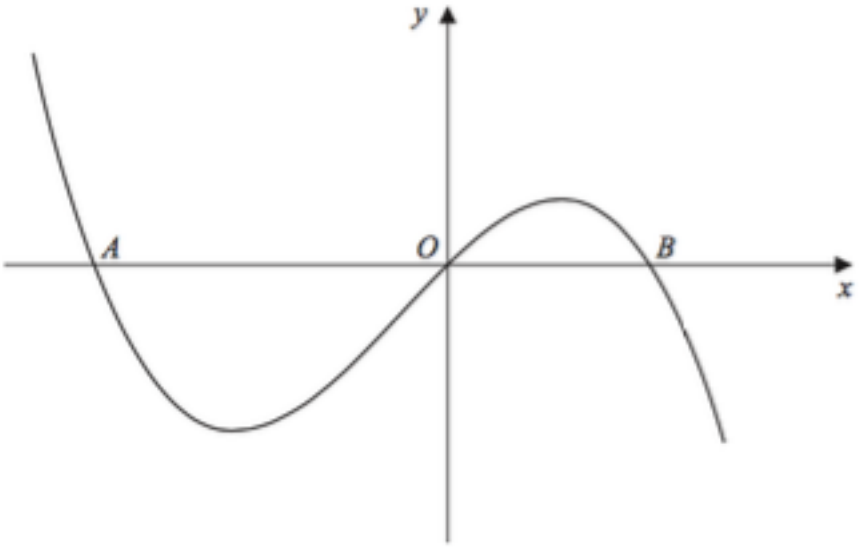


Figure 5

Figure 5 shows a sketch of the curve C with equation $y = f(x)$.
The curve C crosses the x-axis at the origin, O, and at the points A and B as shown in Figure 5.

Given that

$$f'(x) = k - 4x - 3x^2$$

where k is constant,

(a) show that C has a point of inflection at $x = -\frac{2}{3}$ **(3)**

Given also that the distance $AB = 4\sqrt{2}$

(b) find, showing your working, the integer value of k. **(7)**

2.

A liquid is being heated in an oven maintained at a constant temperature of 160 °C. It may be assumed that the rate of increase of the temperature of the liquid at any particular time t minutes is proportional to $160 - \theta$, where θ °C is the temperature of the liquid at that time.

(i) Write down a differential equation connecting θ and t. **[2]**

When the liquid was placed in the oven, its temperature was 20 °C and 5 minutes later its temperature had risen to 65 °C.

(ii) Find the temperature of the liquid, correct to the nearest degree, after another 5 minutes. **[9]**

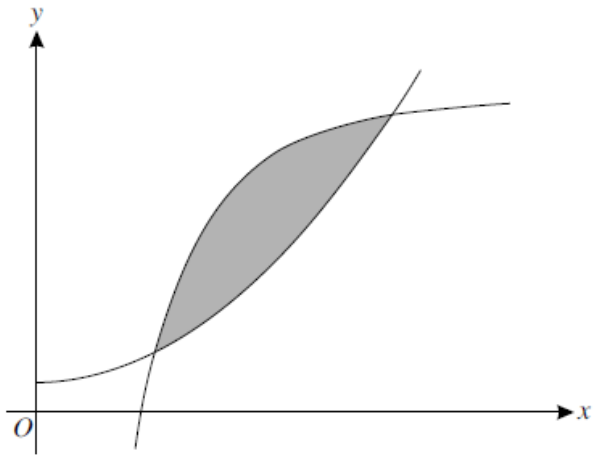
3.

A forest is burning so that, t hours after the start of the fire, the area burnt is A hectares. It is given that, at any instant, the rate at which this area is increasing is proportional to A^2.

(i) Write down a differential equation which models this situation. [2]

(ii) After 1 hour, 1000 hectares have been burnt; after 2 hours, 2000 hectares have been burnt. Find after how many hours 3000 hectares have been burnt. [6]

4.



The diagram shows parts of the curves $y = x^2 + 1$ and $y = 11 - \frac{9}{x^2}$, which intersect at (1, 2) and (3, 10). Use integration to find the exact area of the shaded region enclosed between the two curves. [7]

5.

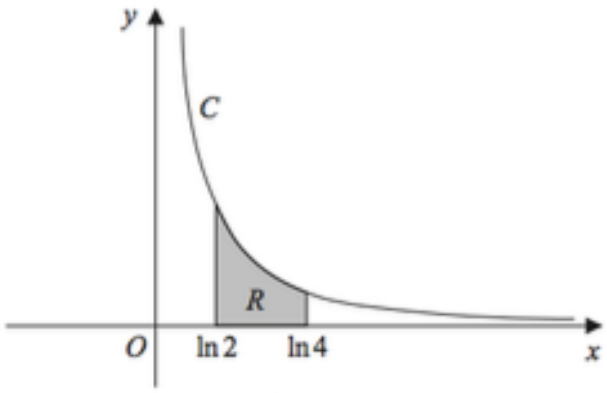


Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = \ln(t + 2), \quad y = \frac{1}{t+1}, \qquad t > -\frac{2}{3}$$

(a) State the domain of values of x for the curve C.

(1)

The finite region R, shown shaded in Figure 4, is bounded by the curve C, the line with equation $x = \ln 2$, the x-axis and the line with equation $x = \ln 4$

(b) Use calculus to show that the area of R is $\ln\left(\frac{3}{2}\right)$.

(8)