Topic X8 Further calculus (Pre-TT A) [45] MARKSCHEME

1.

Question	Scheme	Marks	AOs
11 (a)	$f'(x) = k - 4x - 3x^2$		
	$f''(x) = -4 - 6x = 0$	M1	1.1b
	Criteria 1 **Either** $f''(x) = -4 - 6x = 0 \Rightarrow x = \frac{4}{-6} \Rightarrow x = -\frac{2}{3}$ **or** $f''\left(-\frac{2}{3}\right) = -4 - 6\left(-\frac{2}{3}\right) = 0$ **Criteria 2** **Either** • $f''(-0.7) = -4 - 6(-0.7) = 0.2 > 0$ $f''(-0.6) = -4 - 6(-0.6) = -0.4 < 0$ **or** • $f'''\left(-\frac{2}{3}\right) = -6 \neq 0$		
	At least one of Criteria 1 or Criteria 2	B1	2.4
	Both Criteria 1 and Criteria 2 **and** concludes *C* has a point of inflection at $x = -\frac{2}{3}$	A1	2.1
		(3)	
(b)	$f'(x) = k - 4x - 3x^2$, $AB = 4\sqrt{2}$		
	$f(x) = kx - 2x^2 - x^3 \{+c\}$	M1 A1	1.1b 1.1b
	$f(0) = 0$ or $(0, 0) \Rightarrow c = 0 \Rightarrow f(x) = kx - 2x^2 - x^3$ $\{f(x) = 0 \Rightarrow\}\ f(x) = x(k - 2x - x^2) = 0 \Rightarrow \{x = 0,\}\ k - 2x - x^2 = 0$	A1	2.2a
	$\{x^2 + 2x - k = 0\} \Rightarrow (x+1)^2 - 1 - k = 0$, $x = ...$	M1	2.1
	$\Rightarrow x = -1 \pm \sqrt{k+1}$	A1	1.1b
	$AB = \left(-1 + \sqrt{k+1}\right) - \left(-1 - \sqrt{k+1}\right) = 4\sqrt{2} \Rightarrow k = ...$	M1	2.1
	So, $2\sqrt{k+1} = 4\sqrt{2} \Rightarrow k = 7$	A1	1.1b
		(7)	
			(10 marks)

2.

(i)	$\frac{d\theta}{dt} = \ldots$	B1	
	$k(160-\theta)$	B1 (2)	The 2 @ 'B1' are indep
(ii)	Separate variables with $(160-\theta)$ in denom; or invert	*M1	$\int \frac{1}{160-\theta}\,d\theta = \int k, \frac{1}{k}, 1\,dt$
	Indication that LHS = $\ln f(\theta)$	A1	If wrong ln, final 3@A = 0
	RHS = kt or $\frac{1}{k}t$ or t $(+c)$	A1	
	Subst. $t=0, \theta=20$ into equation containing 'c'	dep* M1	
	Subst $t=5, \theta=65$ into equation containing 'c' & 'k'	dep* M1	
	$c = -\ln 140$ (-4.94) ISW	A1	
	$k = \frac{1}{5}\ln\frac{140}{95}$ (≈ 0.077 or 0.078) ISW	A1	
	Using their 'c' & 'k', subst $t = 10$ & evaluate θ	dep*M1	
	$\theta = 96(95.535714)$ $\left(95\frac{15}{28}\right)$	A1 (9)	
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3.

(i)	$\frac{dA}{dt}$ or kA^2 seen	**M1**		
	$\frac{dA}{dt} = kA^2$	**A1**	**2**	
(ii)	Separate variables + attempt to integrate	***M1**		Accept if based on $\frac{dA}{dt} = kA^2$ or A^2
	$-\frac{1}{A} = kt + c$ or $-\frac{1}{kA} = t + c$ or $-\frac{1}{A} = t + c$	**A1**		
	Subst one of (0,0),(1,1000) or (2,2000) into eqn.	**dep*M1**		Equation must contain k and/or c
	Subst another of (0,),(1,1000) or (2,2000) into eqn	**dep*M1**		This equation must contain k and c
	Substitute $A = 3000$ into eqn with k and c subst	**dep*M1**		
	$t = \frac{7}{3}$ ISW	**A1**	**6**	Accept 2.33, 2h 20 m

4.

$\int_1^3 \{(11-9x^{-2})-(x^2+1)\}dx = \left[9x^{-1}-\frac{1}{3}x^3+10x\right]_1^3$	M1		Attempt subtraction (correct order) at any point
$= (3-9+30)-(9-\frac{1}{3}+10)$	M1		Attempt integration – inc. in power for at least one term
$= 24 - 18\frac{2}{3}$	A1		Obtain $\pm(-\frac{1}{3}x^3+10x)$ or $11x$ and $\frac{1}{3}x^3+x$
$= 5\frac{1}{3}$	M1		Obtain remaining term of form kx^{-1}
OR	A1		Obtain $\pm 9x^{-1}$ or any unsimplified equiv
$\left[11x+9x^{-1}\right]_1^3 - \left[\frac{1}{3}x^3+x\right]_1^3$	M1		Use limits $x = 1, 3$ – correct order & subtraction
$= [(33+3)-(11+9)]-[(9+3)-(\frac{1}{3}+1)]$ $= 16 - 10\frac{2}{3}$ $= 5\frac{1}{3}$	A1	7	Obtain $5\frac{1}{3}$, or exact equiv

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5.

10 (a)	$x > \ln\left(\frac{4}{3}\right)$	B1	2.2a
(b)	Attempts to apply $\int y\frac{dx}{dt}dt$	M1	3.1a
	$\left\{\int y\frac{dx}{dt}dt = \right\} = \int\left(\frac{1}{t+1}\right)\left(\frac{1}{t+2}\right)dt$	A1	1.1b
	$\frac{1}{(t+1)(t+2)} \equiv \frac{A}{(t+1)} + \frac{B}{(t+2)} \Rightarrow 1 \equiv A(t+2) + B(t+1)$	M1	3.1a
	$\{A = 1, B = -1 \Rightarrow\}$ gives $\frac{1}{(t+1)} - \frac{1}{(t+2)}$	A1	1.1b
	$\left\{\int\left(\frac{1}{(t+1)} - \frac{1}{(t+2)}\right)dt = \right\} \ln(t+1) - \ln(t+2)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = \left[\ln(t+1) - \ln(t+2)\right]_0^2 = (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$	M1	2.2a
	$= \ln 3 - \ln 4 + \ln 2 = \ln\left(\frac{(3)(2)}{4}\right) = \ln\left(\frac{6}{4}\right)$		
	$= \ln\left(\frac{3}{2}\right)$ *	A1*	2.1