

# Topic Y5 Probability and proof (Pre-TT) [42] MARKSCHEME

1.

(i)	$\frac{3}{4} + \frac{1}{4} \times \frac{3}{8}$ $+ \frac{1}{4} \times \frac{5}{8} \times \frac{3}{16}$ $= \frac{447}{512} \text{ or } 0.873 \text{ (3 sf)}$	M1 M1 A1 [3]	$\frac{1}{4} \times \frac{5}{8} \times \frac{13}{16} \quad (= \frac{65}{512} \text{ or } 0.127)$ $1 - \frac{1}{4} \times \frac{5}{8} \times \frac{13}{16}$	
(ii)	0.6p or equiv seen $0.4 + 0.6p = 0.58$ $p = 0.3$	B1 M1 A1 [3]	Tree diag alone insufficient for mark. Or $0.6p = 0.18$ . "0.18" alone insufficient	NB $0.6 \times 0.3 = 0.18$ seen at the end is probably a check, not an answer. But if 0.3 seen and 0.18 is <u>very</u> clearly indicated as the ans then B1M1A0

2.

5(i)	$\frac{12}{22} \times \frac{11}{21}$ $= \frac{2}{7} \text{ oe or } 0.286 \text{ (3 sfs)}$	M1 A1 2	or ${}^{12}C_2 / {}^{22}C_2$
(ii)	$\frac{7}{15} \times \frac{6}{14} \times \frac{8}{13} \quad \text{or } \frac{8}{65} \text{ oe}$ $\times 3 \text{ oe}$ $= \frac{24}{65} \text{ or } 0.369 \text{ (3 sfs)}$	M1 M1 A1 3	Numerators any order ${}^7C_2 \times {}^8C_1$ :M1 3 x prod any 3 probs (any C or P)/ ${}^{15}C_3$ :M1 (dep <1)  $1 - (\frac{8}{15} \times \frac{7}{14} \times \frac{6}{13} + 3 \times \frac{8}{15} \times \frac{7}{14} \times \frac{7}{13} + \frac{7}{15} \times \frac{6}{14} \times \frac{5}{13})$ : M2  one prod omitted or wrong: M1
(iii)	$\frac{x}{45} \times \frac{x-1}{44} = \frac{1}{15} \text{ oe}$ $x^2 - x - 132 = 0 \quad \text{or } x(x-1) = 132$ $(x-12)(x+11) = 0$ $\text{or } x = \frac{1 \pm \sqrt{(1^2 - 4 \times (-132))}}{2}$ No. of Ys = 12	M1  A1  M1  A1 4	not $\frac{x}{45} \times \frac{x}{44} = \frac{1}{15}$ or $\frac{x}{45} \times \frac{x}{45} = \frac{1}{15}$ or $\frac{x}{45} \times \frac{x-1}{45} = \frac{1}{15}$  oe  ft 3-term QE for M1 condone signs interchanged allow one sign error  Not $x = 12$ or $-11$ ans 12 from less wking, eg $12 \times 11 = 132$ or T & I: <span style="float: right;">full mks</span>  Some incorrect methods:  $\frac{x}{45} \times \frac{x-1}{44} = \frac{1}{15} \text{ oe}$ M1 $x^2 + x = 132$ A0 $x = 11$ M1A0  $12 \times 11 = 132$ M1A1M1 $x = 12$ and (or "or") 11 A0  NB 12 from eg 12.3 rounded, check method
<b>Total</b>		<b>9</b>	

3.

<b>14 (i)</b>	For an explanation or statement to show when the claim $3^x \geq 2^x$ fails This could be e.g. <ul style="list-style-type: none"> <li>• when <math>x = -1</math>, <math>\frac{1}{3} &lt; \frac{1}{2}</math> or <math>\frac{1}{3}</math> is not greater than or equal to <math>\frac{1}{2}</math></li> <li>• when <math>x &lt; 0</math>, <math>3^x &lt; 2^x</math> or <math>3^x</math> is not greater than or equal to <math>2^x</math></li> </ul>	M1	2.3
	followed by an explanation or statement to show when the claim $3^x \geq 2^x$ is true. This could be e.g. <ul style="list-style-type: none"> <li>• <math>x = 2</math>, <math>9 \geq 4</math> or 9 is greater than or equal to 4</li> <li>• when <math>x \geq 0</math>, <math>3^x \geq 2^x</math></li> </ul> and a correct conclusion. E.g. <ul style="list-style-type: none"> <li>• so the claim <math>3^x \geq 2^x</math> is sometimes true</li> </ul>	A1	2.4

			<b>(2)</b>	
<b>(ii)</b>	Assume that $\sqrt{3}$ is a rational number		M1	2.1
	So $\sqrt{3} = \frac{p}{q}$ , where $p$ and $q$ integers, $q \neq 0$ , and the HCF of $p$ and $q$ is 1			
	$\Rightarrow p = \sqrt{3}q \Rightarrow p^2 = 3q^2$		M1	1.1b
	$\Rightarrow p^2$ is divisible by 3 and so $p$ is divisible by 3		A1	2.2a
	So $p = 3c$ , where $c$ is an integer		M1	2.1
	From earlier, $p^2 = 3q^2 \Rightarrow (3c)^2 = 3q^2$			
	$\Rightarrow q^2 = 3c^2 \Rightarrow q^2$ is divisible by 3 and so $q$ is divisible by 3		A1	1.1b
As both $p$ and $q$ are both divisible by 3 then the HCF of $p$ and $q$ is not 1		A1	2.4	
This contradiction implies that $\sqrt{3}$ is an irrational number				
			<b>(6)</b>	

4.

<b>(i)</b>	Attempt to represent information e.g. by Venn diagram with $x$ in centre and 3 other correct values in terms of $x$	<b>B1</b>	<b>3.3</b>	Any equivalent method	<b>OR</b> <b>B1</b> $\frac{18}{30} + \frac{19}{30} + \frac{17}{30} - \left(\frac{8}{30} + \frac{9}{30} + \frac{11}{30}\right) = \frac{26}{30}$ <b>M1</b> $1 - \frac{26}{30} = \frac{4}{30}$
	Attempt total (in terms of $x$ ) = 30	<b>M1</b>	<b>3.4</b>		
	$x = 4$ so $n(S \cap H \cap T) = 4$	<b>E1</b>	<b>1.1</b>	Or the number doing all three is 4. <b>E0</b> for just $x = 4$	
		<b>[3]</b>			
<b>(ii)</b>	$\frac{5}{9}$ oe	<b>B1FT</b>	<b>2.2a</b>	FT their (i)	
		<b>[1]</b>			
<b>(iii)</b>	$\frac{5}{9} \times \frac{19}{29}$	<b>B1</b>	<b>2.2a</b>	All correct	
	$\frac{4}{9} \times \frac{18}{29}$	<b>B1</b>	<b>2.2a</b>		
	$\frac{5}{9} \times \frac{19}{29} + \frac{4}{9} \times \frac{18}{29}$	<b>M1</b>	<b>2.2a</b>		
	$= \frac{167}{261}$ oe or 0.640 (3 s.f.)	<b>A1</b>	<b>1.1</b>		
		<b>[4]</b>			

5.

<b>7i</b>	$\frac{2}{9}$ or $\frac{7}{9}$ oe seen $\frac{3}{9}$ or $\frac{6}{9}$ oe seen $\frac{1}{8}$ or $\frac{7}{8}$ oe seen Correct structure All correct	B1 B1 B1 B1 B1	5	ie 8 correct branches only, ignore probs & values including probs and values, but headings not req'd
<b>ii</b>	$\frac{3}{10} \times \frac{7}{9} + \frac{7}{10} \times \frac{3}{9} + \frac{7}{10} \times \frac{6}{9}$ $\frac{14}{15}$ or 0.933 oe	M2 A1	3	or $\frac{3}{10} \times \frac{7}{9} + \frac{7}{10}$ or $1 - \frac{3}{10} \times \frac{2}{9}$ M1: one correct prod or any prod + $\frac{7}{10}$ or $\frac{3}{10} \times \frac{2}{9}$
<b>iii</b>	$\frac{3}{10} \times \frac{2}{9} \times \frac{7}{8} + \frac{7}{10} \times \frac{6}{9}$ $\frac{21}{40}$ or 0.525 oe	M2 A1	3	M1: one correct prod cao
	No ft from diag except: with replacement:	(i) structure: B1	(ii) $\frac{91}{100}$ : B2	(iii) 0.553: B2
<b>Total</b>			<b>11</b>	