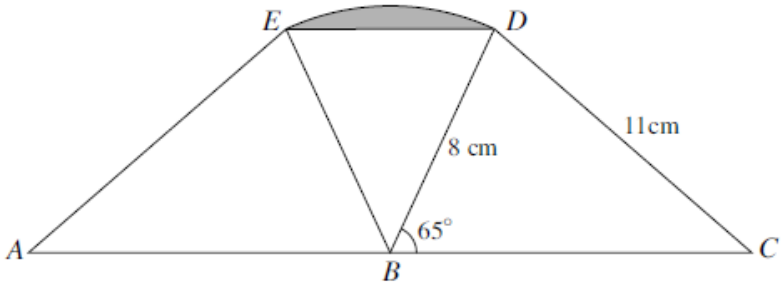
Topic Y6 Further trigonometry (Post-TT A) [41]

1.



The diagram shows two congruent triangles, BCD and BAE, where ABC is a straight line. In triangle BCD, $BD = 8$ cm, $CD = 11$ cm and angle $CBD = 65°$. The points E and D are joined by an arc of a circle with centre B and radius 8 cm.

(i) Find angle BCD. **[2]**

(ii) **(a)** Show that angle EBD is 0.873 radians, correct to 3 significant figures. **[2]**

(b) Hence find the area of the shaded segment bounded by the chord ED and the arc ED, giving your answer correct to 3 significant figures. **[4]**

(Total 8 marks)

2.

Show that, for a small angle θ, where θ is in radians,

$$1+\cos\theta-3\cos^2\theta \approx -1+\tfrac{5}{2}\theta^2 .$$

[4]

(Total 4 marks)

3.

(i) Express $4\cos\theta - 2\sin\theta$ in the form $R\cos(\theta+\alpha)$, where $R > 0$ and $0° < \alpha < 90°$. **[3]**

(ii) Hence

(a) solve the equation $4\cos\theta - 2\sin\theta = 3$ for $0° < \theta < 360°$, **[4]**

(b) determine the greatest and least values of

$$25 - (4\cos\theta - 2\sin\theta)^2$$

as θ varies, and, in each case, find the smallest positive value of θ for which that value occurs. **[5]**

(Total 12 marks)

4.

(a) Prove that

$$\tan\theta + \cot\theta \equiv 2\operatorname{cosec} 2\theta, \qquad \theta \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence explain why the equation

$$\tan\theta + \cot\theta = 1$$

does not have any real solutions.

(1)

(Total 5 marks)

5.

(i) By first expanding $\cos(2\theta + \theta)$, prove that

$$\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta.$$ **[4]**

(ii) Hence prove that

$$\cos 6\theta \equiv 32\cos^6\theta - 48\cos^4\theta + 18\cos^2\theta - 1.$$ **[3]**

(iii) Show that the only solutions of the equation

$$1 + \cos 6\theta = 18\cos^2\theta$$

are odd multiples of 90°. **[5]**

(Total 12 marks)