Topic Y6 Further trigonometry (Post-TT A) [41] MARKSCHEME

1.

5 (i)	$\frac{\sin\theta}{8} = \frac{\sin 65}{11}$	M1		Attempt use of correct sine rule
	$\theta = 41.2°$	A1	2	Obtain 41.2°, or better
(ii) a	180 – (2 x 65) = 50° or 65 x $^{\pi}/_{180}$ = 1.134 50 x $^{\pi}/_{180}$ = 0.873 **A.G.** π – (2 x 1.134) = 0.873	M1		Use conversion factor of $^{\pi}/_{180}$
		A1	2	Show 0.873 radians convincingly (AG)
(ii) b	area sector = ½ x 8^2 x 0.873 = 27.9 area triangle = ½ x 8^2 x sin 0.873 = 24.5	M1		Attempt area of sector, using (½) $r^2\theta$
	area segment = 27.9 – 24.5 = 3.41	M1		Attempt area of triangle using (½) $r^2\sin\theta$
		M1		Subtract area of triangle from area of sector
		A1	4	Obtain 3.41or 3.42
			8	

2.

When θ is small $1+\cos\theta-3\cos^2\theta$ $\approx 1+\left(1-\frac{1}{2}\theta^2\right)-3\left(1-\frac{1}{2}\theta^2\right)^2$	M1	1.1a	Attempt to use $\cos\theta \approx 1-\frac{1}{2}\theta^2$ or $=1+\left(1-\frac{1}{2}\theta^2+...\right)$ $-3\left(1-\frac{1}{2}\theta^2+...\right)^2$	**OR** **M1** Attempt to use $\cos\theta \approx 1-\frac{1}{2}\theta^2$
$=1+\left(1-\frac{1}{2}\theta^2\right)-3\left(1-\theta^2+\frac{1}{4}\theta^4\right)$ $=1+1-\frac{1}{2}\theta^2-3+3\theta^2-\frac{3}{4}\theta^4$	M1	1.1	Multiply out	**M1** use trigonometric identity $1+\cos\theta-3\cos^2\theta$ $=1+\cos\theta-\frac{3}{2}-\frac{3}{2}\cos 2\theta$
Since θ is small, we can neglect the higher order terms	E1	2.5	For explanation of loss of θ^4 term and consistent use of notation throughout (Working need not be fully correct)	**E1** For showing clearly which identity has been used and consistent use of notation throughout
so $1+\cos\theta-3\cos^2\theta \approx -1+\frac{5}{2}\theta^2$ as required	E1	2.1	AG Clearly obtained www Condone θ^4 term missing without explanation and inconsistent notation	**E1** AG Clearly obtained www Condone inconsistent notation
	[4]			

3.

(i)		Obtain $R=\sqrt{20}$ or $R=4.47$	B1	
		Attempt to find value of α	M1	implied by correct value or its complement; allow sin/cos muddles; allow use of radians for M1; condone use of $\cos\alpha=4, \sin\alpha=2$ here but not for A1
		Obtain 26.6	A1 [3]	or greater accuracy 26.565…; with no wrong working seen
(ii)	(a)	Show correct process for finding one answer	M1	allowing for case where the answer is negative
		Obtain 21.3	A1FT	or greater accuracy 21.3045…; or anything rounding to 21.3 with no obvious error; following a wrong value of α but not wrong R
		Show correct process for finding second answer	M1	ie attempting fourth quadrant value minus α value
		Obtain 286 or 285.6	A1FT [4]	or greater accuracy 285.5653…; or anything rounding to 286 with no obvious error; following a wrong value of α but not wrong R ;and no others between 0° and 360°
(ii)	(b)	State greatest value is 25	B1	allow if α incorrect
		Obtain value 63.4 clearly associated with correct greatest value	B1FT	or greater accuracy 63.4349…; following a wrong value of α
		State least value is 5	B1	allow if α incorrect
		Attempt to find θ from $\cos(\theta+\text{their}\alpha)=-1$	M1	and clearly associated with correct least value
		Obtain 153 or 153.4	A1FT [5]	or greater accuracy 153.4349…; following a wrong value of α

4.

9(a)	$\tan\theta+\cot\theta\equiv\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}$	M1	2.1
	$\equiv\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$	A1	1.1b
	$\equiv\dfrac{1}{\frac{1}{2}\sin 2\theta}$	M1	2.1
	$\equiv 2\operatorname{cosec}2\theta$ *	A1*	1.1b
		(4)	
(b)	States $\tan\theta+\cot\theta=1\Rightarrow\sin 2\theta=2$ AND no real solutions as $-1\leqslant\sin 2\theta\leqslant 1$	B1	2.4
		(1)	
			(5 marks)

5.

(i)	State $\cos 2\theta\cos\theta-\sin 2\theta\sin\theta$	B1		
	Use at least one of $\cos 2\theta=2\cos^2\theta-1$ and $\sin 2\theta=2\sin\theta\cos\theta$	B1		
	Attempt to express in terms of $\cos\theta$ only	M1		using correct identities for $\cos 2\theta$, $\sin 2\theta$ and $\sin^2\theta$
	Obtain $4\cos^3\theta-3\cos\theta$	A1	**4**	AG; necessary detail required
(ii)	Either: State or imply $\cos 6\theta=2\cos^2 3\theta-1$	B1		
	Use expression for $\cos 3\theta$ and attempt expansion	M1		for expression of form $\pm 2\cos^2 3\theta\pm 1$
	Obtain $32c^6-48c^4+18c^2-1$	A1	**3**	AG; necessary detail required
	Or: State $\cos 6\theta=4\cos^3 2\theta-3\cos 2\theta$	B1		maybe implied
	Express $\cos 2\theta$ in terms of $\cos\theta$ and attempt expansion	M1		for expression of form $\pm 2\cos^2\theta\pm 1$
	Obtain $32c^6-48c^4+18c^2-1$	A1	**(3)**	AG; necessary detail required
(iii)	Substitute for $\cos 6\theta$	*M1		with simplification attempted
	Obtain $32c^6-48c^4=0$	A1		or equiv
	Attempt solution for c of equation	M1		dep *M
	Obtain $c^2=\frac{3}{2}$ and observe no solutions	A1		or equiv; correct work only
	Obtain $c=0$, give at least three specific angles and conclude odd multiples of 90	A1	**5**	AG; or equiv; necessary detail required; correct work only
			12	