Topic Y6 Further trigonometry (Post-TT B) [51]

1.

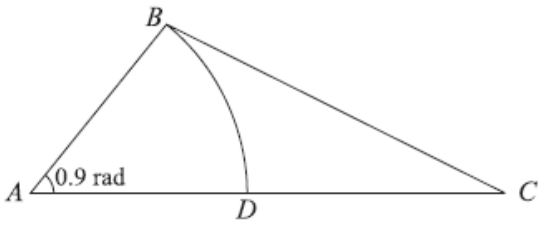
(i) Express $2\tan^2\theta - \dfrac{1}{\cos\theta}$ in terms of $\sec\theta$. **[3]**

(ii) Hence solve, for $0° < \theta < 360°$, the equation

$$2\tan^2\theta - \frac{1}{\cos\theta} = 4.$$ **[4]**

(Total 7 marks)

2.



The diagram shows a triangle ABC, where angle BAC is 0.9 radians. BAD is a sector of the circle with centre A and radius AB.

(i) The area of the sector BAD is 16.2 cm^2. Show that the length of AB is 6 cm. [2]

(ii) The area of triangle ABC is twice the area of sector BAD. Find the length of AC. [3]

(iii) Find the perimeter of the region BCD. [6]

(Total 11 marks)

3.

$f(x) = \sin x$

Using differentiation from first principles find the exact value of $f'\left(\frac{\pi}{6}\right)$

Fully justify your answer.

(Total 6 marks)

4.

(i) Show that the equation

$$2\sin x\tan x - 5 = \cos x$$

can be expressed in the form

$$3\cos^2 x + 5\cos x - 2 = 0.$$ [3]

(ii) Hence solve the equation

$$2\sin x\tan x - 5 = \cos x,$$

giving all values of x, in radians, for $0 \leqslant x \leqslant 2\pi$. [4]

(Total 6 marks)

5.

(a) Express $10\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$
Give the exact value of R and give the value of α, in degrees, to 2 decimal places.

(3)

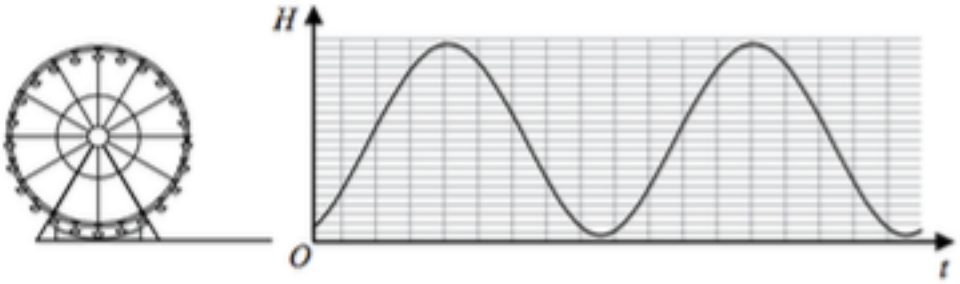


Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation

$$H = a - 10\cos(80t)° + 3\sin(80t)°$$

where a is a constant.

Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(b) (i) find a complete equation for the model,

(ii) hence find the maximum height of the passenger above the ground.

(2)

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(3)

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed?

(1)

(Total 9 marks)

6.

(i) Prove that $\dfrac{\sin(\theta-\alpha)+3\sin\theta+\sin(\theta+\alpha)}{\cos(\theta-\alpha)+3\cos\theta+\cos(\theta+\alpha)} \equiv \tan\theta$ for all values of α. **[5]**

(ii) Find the exact value of $\dfrac{4\sin 149° + 12\sin 150° + 4\sin 151°}{3\cos 149° + 9\cos 150° + 3\cos 151°}$. **[3]**

(iii) It is given that k is a positive constant. Solve, for $0° < \theta < 60°$ and in terms of k, the equation

$$\frac{\sin(6\theta-15°)+3\sin 6\theta+\sin(6\theta+15°)}{\cos(6\theta-15°)+3\cos 6\theta+\cos(6\theta+15°)} = k.$$ **[4]**

(Total 12 marks)