Topic Y6 Further trigonometry (Post-TT B) [51] MARKSCHEME

1.

(i)	Attempt use of identity for $\tan^2\theta$	M1		using $\pm\sec^2\theta \pm 1$; or equiv
	Replace $\dfrac{1}{\cos\theta}$ by $\sec\theta$	B1		
	Obtain $2(\sec^2\theta - 1) - \sec\theta$	A1	3	or equiv
(ii)	Attempt soln of quadratic in $\sec\theta$ or $\cos\theta$	M1		as far as factorisation or substitution in correct formula
	Relate $\sec\theta$ to $\cos\theta$ and attempt at least one value of θ	M1		may be implied
	Obtain $60°, 131.8°$	A1		allow 132 or greater accuracy
	Obtain $60°, 131.8°, 228.2°, 300°$	A1	4	allow 132, 228 or greater accuracy; and no others between $0°$ and $360°$
			7	

2.

(i)	$\frac{1}{2}\times AB^2 \times 0.9 = 16.2$	M1	Use $(\frac{1}{2})r^2\theta = 16.2$
	$AB^2 = 36 \Rightarrow AB = 6$	A1 2	Confirm *AB* = 6 cm (or verify ½ x 6^2 x 0.9 = 16.2)
(ii)	$\frac{1}{2}\times 6 \times AC \times \sin 0.9 = 32.4$	M1*	Use $\Delta = \frac{1}{2}bc\sin A$, or equiv
		M1dep*	Equate attempt at area to 32.4
	AC = 13.8 cm	A1 3	Obtain *AC* = 13.8 cm, or better
(iii)	$BC^2 = 6^2 + 13.8^2 - 2\times 6\times 13.8\times\cos 0.9$	M1	Attempt use of correct cosine formula in ΔABC
		A1√	Correct unsimplified equation, from their *AC*
	Hence *BC* = 11.1 cm	A1	Obtain *BC* = 11.1 cm, or anything that rounds to this
	$BD = 6\times 0.9 = 5.4$ cm	B1	State *BD* = 5.4 cm (seen anywhere in question)
	Hence perimeter = 11.1 + 5.4 + (13.8 – 6)	M1	Attempt perimeter of region *BCD*
	= 24.3 cm	A1 6	Obtain 24.3 cm, or anything that rounds to this
		11	

3.

(a) (i)	$2\sin x.\frac{\sin x}{\cos x} - 5 = \cos x$	M1	Use $\tan x \equiv \frac{\sin x}{\cos x}$
	$2\sin^2 x - 5\cos x = \cos^2 x$		
	$2 - 2\cos^2 x - 5\cos x = \cos^2 x$	M1	Use $\sin^2 x \equiv 1 - \cos^2 x$
	$3\cos^2 x + 5\cos x - 2 = 0$	A1	Show given equation convincingly
		3	
(ii)	$(3\cos x - 1)(\cos x + 2) = 0$	M1	Attempt to solve quadratic in $\cos x$
	$\cos x = {}^1/_3$	M1	Attempt to find *x* from root(s) of quadratic
	$x = 1.23$ rad	A1	Obtain 1.23 rad or 70.5°
	$x = 5.05$ rad	A1√	Obtain 5.05 rad or 289° (or 2π / 360° - their solution)
			SR: B1 B1 for answer(s) only
		4	

4.

Translates $f'\left(\frac{\pi}{6}\right)$ into $\lim_{h\to 0}\frac{\sin\left(\frac{\pi}{6}+h\right)-\sin\left(\frac{\pi}{6}\right)}{h}$	AO1.1a	M1	$f'\left(\frac{\pi}{6}\right)=\lim_{h\to 0}\left[\frac{\sin\left(\frac{\pi}{6}+h\right)-\sin\left(\frac{\pi}{6}\right)}{h}\right]$
Uses sin ($A + B$) identity to replace $\sin\left(\frac{\pi}{6}+h\right)$, to commence argument (at least two lines of argument seen)	AO2.1	M1	$=\lim_{h\to 0}\left[\frac{\sin\frac{\pi}{6}\cos h+\cos\frac{\pi}{6}\sin h-\sin\left(\frac{\pi}{6}\right)}{h}\right]$ $=\lim_{h\to 0}\left[\frac{\frac{1}{2}\cos h+\frac{\sqrt{3}}{2}\sin h-\frac{1}{2}}{h}\right]$
Obtains correct two term expression involving cos h and sin h	AO1.1b	A1	$=\lim_{h\to 0}\left[\frac{1}{2}\left(\frac{\cos h-1}{h}\right)+\frac{\sqrt{3}}{2}\frac{\sin h}{h}\right]$
Deduce what happens as $h\to 0$, for one part of 'their' expression using the limit of $\frac{\sin h}{h}$ OR by using small angles approximations	AO2.2a	R1	$=\lim_{h\to 0}\left[\frac{1}{2}\left(\frac{-2\sin^2\left(\frac{h}{2}\right)}{\frac{2h}{2}}\right)+\frac{\sqrt{3}}{2}\frac{\sin h}{h}\right]$ $=\left(\lim_{h\to 0}\frac{-\sin\left(\frac{h}{2}\right)}{2}\right)\left(\lim_{h\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\right)+\frac{\sqrt{3}}{2}\lim_{h\to 0}\frac{\sin h}{h}$
Deduce what happens as $h\to 0$, for the second part of 'their' expression using the limit of (cos h – 1)h OR by using small angle approximations	AO2.2a	R1	$=0\times 1+\frac{\sqrt{3}}{2}\times 1$ $=\frac{\sqrt{3}}{2}$
Completes a rigorous argument leading to the correct exact value, with all the steps in the method clearly shown.	AO2.1	R1	
Total		**6**	

5.

13(a)	$R=\sqrt{109}$	B1	1.1b
	$\tan\alpha=\dfrac{3}{10}$	M1	1.1b
	$\alpha=16.70°$ so $\sqrt{109}\cos(\theta+16.70°)$	A1	1.1b
		(3)	
(b)	(i) e.g $H=11-10\cos(80t)°+3\sin(80t)°$ or $H=11-\sqrt{109}\cos(80t+16.70)°$	B1	3.3
	(ii) $11+\sqrt{109}$ or 21.44 m	B1ft	3.4
		(2)	
(c)	Sets $80t+"16.70"=540$	M1	3.4
	$t=\dfrac{540-"16.70"}{80}=(6.54)$	M1	1.1b
	$t=6$ mins 32 seconds	A1	1.1b
		(3)	
(d)	Increase the '80' in the formula For example use $H=11-10\cos(90t)°+3\sin(90t)°$		3.3
		(1)	
			(9 marks)

6.

(i)	Use at least one identity correctly	B1		angle-sum or angle-difference identity
	Attempt use of relevant identities in single rational expression	M1		not earned if identities used in expression where step equiv to $\dfrac{A+B+C}{D+E+F}=\dfrac{A}{D}+\dfrac{B}{E}+\dfrac{C}{F}$ or similar has been carried out; condone (for M1A1) if signs of identities apparently switched (so that, for example, denominator appears as $\cos\theta\cos\alpha-\sin\theta\sin\alpha+3\cos\theta+\cos\theta\cos\alpha+\sin\theta\sin\alpha$)
	Obtain $\dfrac{2\sin\theta\cos\alpha+3\sin\theta}{2\cos\theta\cos\alpha+3\cos\theta}$	A1		or equiv but with the other two terms from each of num'r and den'r absent
	Attempt factorisation of num'r and den'r	M1		
	Obtain $\dfrac{\sin\theta}{\cos\theta}$ and hence $\tan\theta$	A1	5	AG; necessary detail needed
(ii)	State or imply form $k\tan 150°$	M1		obtained without any wrong method seen
	State or imply $\frac{4}{3}\tan 150°$	A1		or equiv such as $\dfrac{12\sin 150°}{9\cos 150°}$
	Obtain $-\frac{4}{9}\sqrt{3}$	A1	3	or exact equiv (such as $-\frac{4}{3\sqrt{3}}$); correct answer only earns 3/3
(iii)	State or imply $\tan 6\theta=k$	B1		
	State $\frac{1}{6}\tan^{-1}k$	B1		
	Attempt second value of θ	M1		using $6\theta=\tan^{-1}k+$ (multiple of 180)
	Obtain $\frac{1}{6}\tan^{-1}k+30°$	A1	4	and no other value

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