Topic Y6 Further trigonometry (Pre-TT A) [47]

1.

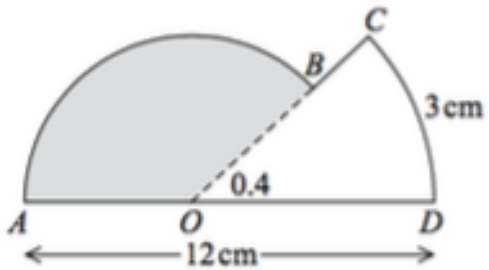


Figure 1

The shape $ABCDOA$, as shown in Figure 1, consists of a sector COD of a circle centre O joined to a sector AOB of a different circle, also centre O.

Given that arc length $CD = 3$ cm, $\angle COD = 0.4$ radians and AOD is a straight line of length 12 cm,

(a) find the length of OD,

(2)

(b) find the area of the shaded sector AOB.

(3)

(Total 5 marks)

2.

(a) Given that θ is small, use the small angle approximation for $\cos\theta$ to show that

$$1 + 4\cos\theta + 3\cos^2\theta \approx 8 - 5\theta^2$$

(3)

Adele uses $\theta = 5°$ to test the approximation in part (a).

Adele's working is shown below.

> Using my calculator, $1 + 4\cos(5°) + 3\cos^2(5°) = 7.962$, to 3 decimal places.
>
> Using the approximation $8 - 5\theta^2$ gives $8 - 5(5)^2 = -117$
>
> Therefore, $1 + 4\cos\theta + 3\cos^2\theta \approx 8 - 5\theta^2$ is not true for $\theta = 5°$

(b) (i) Identify the mistake made by Adele in her working.

(ii) Show that $8 - 5\theta^2$ can be used to give a good approximation to $1 + 4\cos\theta + 3\cos^2\theta$ for an angle of size 5°

(2)

(Total 5 marks)

3.

(a) Given that $7\sin 2\alpha = 3\sin\alpha$, where $0° < \alpha < 90°$, find the exact value of $\cos\alpha$. **[3]**

(b) Given that $3\cos 2\beta + 19\cos\beta + 13 = 0$, where $90° < \beta < 180°$, find the exact value of $\sec\beta$. **[5]**

(Total 8 marks)

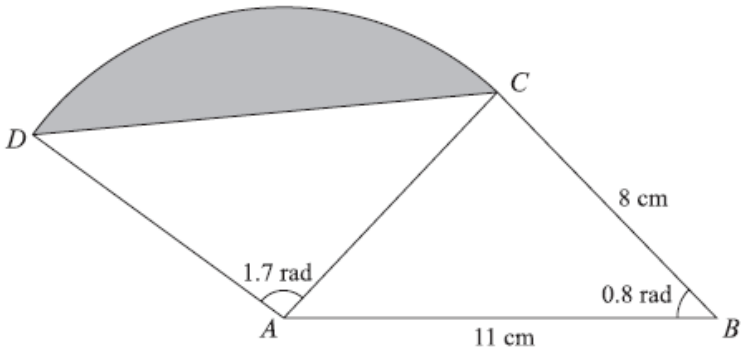
4.

(i) Express $3\sin\theta + 2\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(ii) Hence solve the equation $3\sin\theta + 2\cos\theta = \frac{7}{2}$, giving all solutions for which $0° < \theta < 360°$. [5]

(Total 8 marks)

5.



The diagram shows a triangle ABC, and a sector ACD of a circle with centre A. It is given that $AB = 11$ cm, $BC = 8$ cm, angle $ABC = 0.8$ radians and angle $DAC = 1.7$ radians. The shaded segment is bounded by the line DC and the arc DC.

(i) Show that the length of AC is 7.90 cm, correct to 3 significant figures. [3]

(ii) Find the area of the shaded segment. [3]

(iii) Find the perimeter of the shaded segment. [4]

(Total 10 marks)

6.

(a) Express $2\sin\theta - 1.5\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

State the value of R and give the value of α to 4 decimal places.

(3)

Tom models the depth of water, D metres, at Southview harbour on 18th October 2017 by the formula

$$D = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right), \quad 0 \leqslant t \leqslant 24$$

where t is the time, in hours, after 00:00 hours on 18th October 2017.

Use Tom's model to

(b) find the depth of water at 00:00 hours on 18th October 2017,

(1)

(c) find the maximum depth of water,

(1)

(d) find the time, in the afternoon, when the maximum depth of water occurs. Give your answer to the nearest minute.

(3)

(cont)

Tom's model is supported by measurements of D taken at regular intervals on 18th October 2017. Jolene attempts to use a similar model in order to model the depth of water at Southview harbour on 19th October 2017.

Jolene models the depth of water, H metres, at Southview harbour on 19th October 2017 by the formula

$$H = 6 + 2\sin\left(\frac{4\pi x}{25}\right) - 1.5\cos\left(\frac{4\pi x}{25}\right), \quad 0 \leqslant x \leqslant 24$$

where x is the time, in hours, after 00:00 hours on 19th October 2017.

By considering the depth of water at 00:00 hours on 19th October 2017 for both models,

(e) (i) explain why Jolene's model is not correct,

(ii) hence find a suitable model for H in terms of x.

(3)

(Total 11 marks)