Topic Y6 Further trigonometry (Pre-TT A) [47] MARKSCHEME

1.

Question	Scheme	Marks	AOs
2(a)	Uses $s = r\theta \Rightarrow 3 = r \times 0.4$	M1	1.2
	$\Rightarrow OD = 7.5$ cm	A1	1.1b
		(2)	
(b)	Uses angle $AOB = (\pi - 0.4)$ or uses radius is (12 – '7.5') cm	M1	3.1a
	Uses area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2} \times (12-7.5)^2 \times (\pi - 0.4)$	M1	1.1b
	$= 27.8\text{cm}^2$	A1ft	1.1b
		(3)	
			(5 marks)

2.

Question	Scheme	Marks	AOs
2(a)	Attempts to substitute $\cos\theta \approx 1 - \frac{1}{2}\theta^2$ into either $1 + 4\cos\theta$ or $3\cos^2\theta$	M1	1.1b
	$1 + 4\cos\theta + 3\cos^2\theta \approx 1 + 4\left(1 - \frac{1}{2}\theta^2\right) + 3\left(1 - \frac{1}{2}\theta^2\right)^2$		
	$= 1 + 4\left(1 - \frac{1}{2}\theta^2\right) + 3\left(1 - \theta^2 + \frac{1}{4}\theta^4\right)$	M1	1.1b
	$= 1 + 4 - 2\theta^2 + 3 - 3\theta^2 + \frac{3}{4}\theta^4$		
	$= 8 - 5\theta^2$ *	A1*	2.1
		(3)	
(b)(i)	E.g. • Adele is working in degrees and not radians • Adele should substitute $\theta = \frac{5\pi}{180}$ and not $\theta = 5$ into the approximation	B1	2.3
(b)(ii)	$8 - 5\left(\frac{5\pi}{180}\right)^2 =$ awrt 7.962, so $\theta = 5°$ gives a good approximation.	B1	2.4
		(2)	
			(5 marks)

3.

(a)	State $14\sin\alpha\cos\alpha = 3\sin\alpha$	B1		or unsimplified equiv such as $7(2\sin\alpha\cos\alpha) = 3\sin\alpha$
	Attempt to find value of $\cos\alpha$	M1		by valid process; may be implied
	Obtain $\frac{3}{14}$	A1	3	exact answer required; ignore subsequent work to find angle
(b)	Attempt use of identity for $\cos 2\beta$	M1		of form $\pm 2\cos^2\beta \pm 1$; initial use of $\cos^2\beta - \sin^2\beta$ needs attempt to express $\sin^2\beta$ in terms of $\cos^2\beta$ to earn M1
	Obtain $6\cos^2\beta + 19\cos\beta + 10$	A1		or unsimplified equiv or equiv involving $\sec\beta$
	Attempt solution of 3-term quadratic eqn	M1		for $\cos\beta$ or (after adjustment) for $\sec\beta$
	Use $\sec\beta = \dfrac{1}{\cos\beta}$ at some stage	M1		or equiv
	Obtain $-\frac{3}{2}$	A1	5	or equiv; and (finally) no other answer
			8	

4.

(i)	Obtain $R = \sqrt{13}$, or 3.6 or 3.61 or greater accuracy	**B1**	
	Attempt recognisable process for finding α	**M1**	[allow sine/cosine muddles]
	Obtain $\alpha = 33.7$	**A1**	**3** [or greater accuracy]
(ii)	Attempt to find at least one value of $\theta + \alpha$	***M1**	
	Obtain value rounding to 76 or 104	**A1√**	[following their R]
	Subtract their α from at least one value	**M1**	[dependent on ***M**]
	Obtain one value rounding to 42 or 43, or to 70	**A1**	
	Obtain other value 42.4 or 70.2	**A1**	**5** [or greater accuracy; no other answers between 0 and 360; ignore answers outside 0 to 360]

5.

(i)	$AC^2 = 11^2 + 8^2 - 2\times 11\times 8\times\cos 0.8$	M1		Attempt to use the cosine formula
	$= 62.3796...$	A1		Correct unsimplified expression
	Hence AC=7.90 cm	A1	3	Show the given answer correctly
(ii)	Area of sector = $\frac{1}{2}\times 7.90^2\times 1.7 = 53.0$	M1		Attempt area of sector using $(\frac{1}{2})r^2\theta$
	Area of triangle = $\frac{1}{2}\times 7.90^2\times\sin 1.7 = 30.9$	M1		Attempt area of ΔACD, using $(\frac{1}{2})r^2\sin\theta$, or equiv
	Hence shaded area = 22.1 cm^2	A1	3	Obtain 22.1
(iii)	(arc) $DC = 7.90\times 1.7 = 13.4$	M1		Use $r\theta$ to attempt arc length
		A1		Obtain 13.4
	(line) $DC^2 = 7.90^2 + 7.90^2 - 2\times 7.90\times 7.90\times\cos 1.7$	M1		Attempt length of line DC using cosine rule or equiv.
	$DC = 11.9$			
	Hence perimeter = 25.3cm	A1	4	Obtain 25.3
			10	

6.

Question	Scheme	Marks	AOs
13 (a)	$R = 2.5$	B1	1.1b
	$\tan\alpha = \dfrac{1.5}{2}$ o.e.	M1	1.1b
	$\alpha = 0.6435,$ so $2.5\sin(\theta - 0.6435)$	A1	1.1b
		(3)	
(b)	e.g. $D = 6 + 2\sin\left(\dfrac{4\pi(0)}{25}\right) - 1.5\cos\left(\dfrac{4\pi(0)}{25}\right) = 4.5\text{m}$ or $D = 6 + 2.5\sin\left(\dfrac{4\pi(0)}{25} - 0.6435\right) = 4.5\text{m}$	B1	3.4
		(1)	
(c)	$D_{\max} = 6 + 2.5 = 8.5$ m	B1ft	3.4
		(1)	
(d)	Sets $\dfrac{4\pi t}{25} - \text{"0.6435"} = \dfrac{5\pi}{2}$ or $\dfrac{\pi}{2}$	M1	1.1b
	Afternoon solution $\Rightarrow \dfrac{4\pi t}{25} - \text{"0.6435"} = \dfrac{5\pi}{2} \Rightarrow t = \dfrac{25}{4\pi}\left(\dfrac{5\pi}{2} + \text{"0.6435"}\right)$	M1	3.1b
	$\Rightarrow t = 16.9052... \Rightarrow \text{Time} = 16:54$ or 4:54 pm	A1	3.2a
		(3)	
(e)(i)	• An attempt to find the depth of water at 00:00 on 19th October 2017 for at least one of either Tom's model or Jolene's model.	M1	3.4
	• At 00:00 on 19th October 2017, Tom: $D = 3.72...$ m and Jolene: $H = 4.5$ m and e.g. • As $4.5 \neq 3.72$ then Jolene's model is not true • Jolene's model is not continuous at 00:00 on 19th October 2017 • Jolene's model does not continue on from where Tom's model has ended	A1	3.5a
(ii)	To make the model continuous, e.g. • $H = 5.22 + 2\sin\left(\dfrac{4\pi x}{25}\right) - 1.5\cos\left(\dfrac{4\pi x}{25}\right), \quad 0 \leqslant x < 24$ • $H = 6 + 2\sin\left(\dfrac{4\pi(x+24)}{25}\right) - 1.5\cos\left(\dfrac{4\pi(x+24)}{25}\right), \quad 0 \leqslant x < 24$	B1	3.3
		(3)	
			(11 marks)