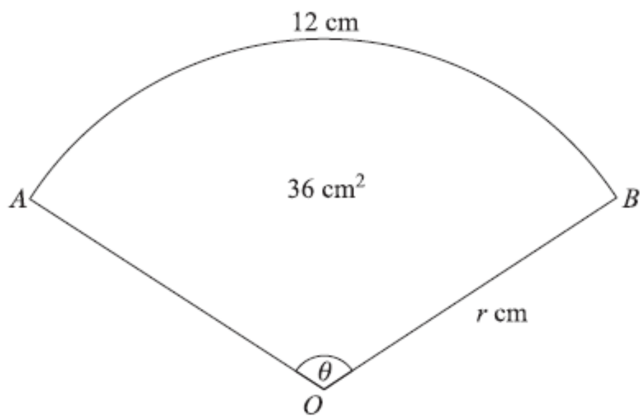
Topic Y6 Further trigonometry (Pre-TT B) [45]

1.



A sector OAB of a circle of radius r cm has angle θ radians. The length of the arc of the sector is 12 cm and the area of the sector is 36 cm^2 (see diagram).

(i) Write down two equations involving r and θ. [2]

(ii) Hence show that $r = 6$, and state the value of θ. [2]

(iii) Find the area of the segment bounded by the arc AB and the chord AB. [3]

(Total 7 marks)

2.

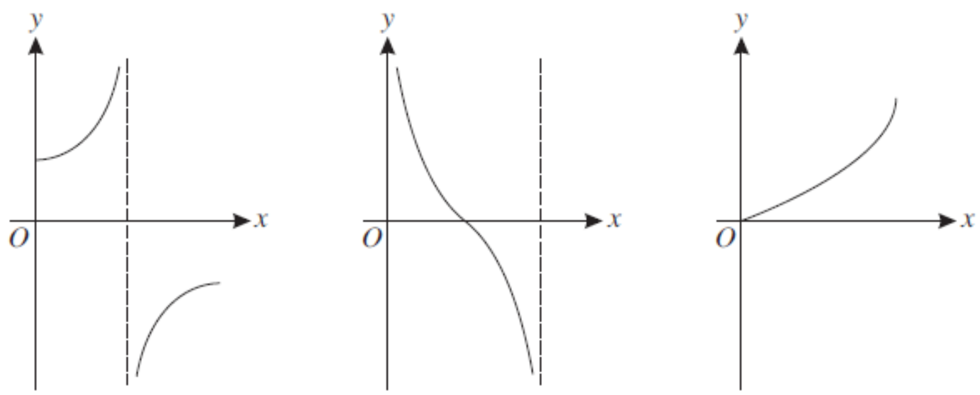


Fig. 1 **Fig. 2** **Fig. 3**

Each diagram above shows part of a curve, the equation of which is one of the following:

$$y = \sin^{-1} x, \quad y = \cos^{-1} x, \quad y = \tan^{-1} x, \quad y = \sec x, \quad y = \operatorname{cosec} x, \quad y = \cot x.$$

State which equation corresponds to

(i) Fig. 1, **[1]**

(ii) Fig. 2, **[1]**

(iii) Fig. 3. **[1]**

(Total 3 marks)

3.

(i) Write down the formula for $\cos 2x$ in terms of $\cos x$. [1]

(ii) Prove the identity $\frac{4\cos 2x}{1+\cos 2x} \equiv 4 - 2\sec^2 x$. [3]

(iii) Solve, for $0 < x < 2\pi$, the equation $\frac{4\cos 2x}{1+\cos 2x} = 3\tan x - 7$. [5]

(Total 9 marks)

4.

10. Given that θ is measured in radians, prove, from first principles, that the derivative of $\sin\theta$ is $\cos\theta$

You may assume the formula for $\sin(A \pm B)$ and that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

(5)

(Total 5 marks)

5.

(i) Express $3\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. **[3]**

(ii) The expression $\mathrm{T}(x)$ is defined by $\mathrm{T}(x) = \frac{8}{3\cos x + 3\sin x}$.

(a) Determine a value of x for which $\mathrm{T}(x)$ is not defined. **[2]**

(b) Find the smallest positive value of x satisfying $\mathrm{T}(3x) = \frac{8}{9}\sqrt{6}$, giving your answer in an exact form. **[4]**

(Total 9 marks)

6.

(i) The polynomial $\mathrm{f}(x)$ is defined by

$$\mathrm{f}(x) = x^3 - x^2 - 3x + 3.$$

Show that $x = 1$ is a root of the equation $\mathrm{f}(x) = 0$, and hence find the other two roots. **[6]**

(ii) Hence solve the equation

$$\tan^3 x - \tan^2 x - 3\tan x + 3 = 0$$

for $0 \leqslant x \leqslant 2\pi$. Give each solution for x in an exact form. **[6]**

(Total 12 marks)