Topic Y6 Further trigonometry (Pre-TT B) [45] MARKSCHEME

1.

(i)	$r\theta = 12$, $\frac{1}{2}r^2\theta = 36$	B1 B1 2	For $r\theta = 12$ stated correctly at any point For $\frac{1}{2}r^2\theta = 36$ stated correctly at any point
(ii)	$\frac{1}{2}r \times 12 = 36 \Rightarrow r = 6$ Hence $\theta = 2$	B1 B1 2	For showing given value correctly For correct value 2 (or 0.637π)
(iii)	Segment area is $36 - \frac{1}{2} \times 6^2 \times \sin 2 = 19.6$ cm^2	M1* M1dep* A1 3 7	For use of $\Delta = \frac{1}{2}ab\sin C$, or equivalent For attempt at $36 - \Delta$ For correct value (rounding to) 19.6

2.

(i)	State $y = \sec x$	B1
(ii)	State $y = \cot x$	B1
(iii)	State $y = \sin^{-1} x$	B1 3
		3

3.

(i)	State $2\cos^2 x - 1$	**B1**	**1**
(ii)	Attempt to express left hand side in terms of $\cos x$ Identify $\frac{1}{\cos x}$ as $\sec x$ Confirm result	**M1** **M1** **A1**	[using expression of form $a\cos^2 x + b$] [maybe implied] **3** [**AG**; necessary detail required]
(iii)	Use identity $\sec^2 x = 1 + \tan^2 x$ Attempt solution of quadratic equation in tan x Obtain $2\tan^2 x + 3\tan x - 9 = 0$ and hence tan $x = -3, \frac{3}{2}$ Obtain at least two of 0.983, 4.12, 1.89, 5.03 (or of 0.313π, 1.31π, 0.602π, 1.60π) Obtain all four solutions	**B1** **M1** **A1** **A1** **A1**	 [or equiv] [allow answers with only 2 s.f.; allow greater accuracy; allow $0.983 + \pi$, $1.89 + \pi$ allow degrees: 56, 236, 108, 288] **5** [now with at least 3 s.f.; must be radians; no other solutions in the range 0 - 2π; ignore solutions outside range 0 - 2π]

4.

10	Use of $\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}$	B1	2.1
	Uses the compound angle identity for $\sin(A+B)$ with $A=\theta,\ B=h$ $\Rightarrow \sin(\theta+h)=\sin\theta\cos h+\cos\theta\sin h$	M1	1.1b
	Achieves $\dfrac{\sin(\theta+h)-\sin\theta}{h}=\dfrac{\sin\theta\cos h+\cos\theta\sin h-\sin\theta}{h}$	A1	1.1b
	$=\dfrac{\sin h}{h}\cos\theta+\left(\dfrac{\cos h-1}{h}\right)\sin\theta$	M1	2.1
	Uses $h\to 0$, $\dfrac{\sin h}{h}\to 1$ and $\dfrac{\cos h-1}{h}\to 0$ Hence the $\text{limit}_{h\to 0}\dfrac{\sin(\theta+h)-\sin\theta}{(\theta+h)-\theta}=\cos\theta$ and the gradient of the chord $\to$ gradient of the curve $\Rightarrow \dfrac{dy}{d\theta}=\cos\theta$ *	A1*	2.5
			(5 marks)

5.

(i) Obtain $R=3\sqrt{2}$ or $R=\sqrt{18}$ or $R=4.24$	B1		or equiv
Attempt to find value of α	M1		condone sin/cos muddles and degrees
Obtain $\frac{1}{4}\pi$ or 0.785	A1	**3**	in radians now
(ii) a Equate $x-\alpha$ to $\frac{1}{2}\pi$ or attempt solution of $3\cos x+3\sin x=0$	M1		condone degrees here
Obtain $\frac{3}{4}\pi$	A1	**2**	or ..., $-\frac{5}{4}\pi, -\frac{1}{4}\pi, \frac{7}{4}\pi, \ldots$; in radians now
b Attempt correct process to find value of $3x-\alpha$	*M1		with attempt at rearranging $T(3x)=\frac{8}{9}\sqrt{6}$
Obtain at least one correct exact value of $3x-\alpha$	A1		$\pm\frac{1}{6}\pi, \pm\frac{11}{6}\pi, \ldots$
Attempt at least one positive value of x	M1		dep *M
Obtain $\frac{1}{36}\pi$	A1	**4**	
	9		

6.

(i) $f(1)=1-1-3+3=0$ **A.G.**	B1		Confirm $f(1)=0$, or division with no remainder shown, or matching coeffs with $R=0$
$f(x)=(x-1)(x^2-3)$	M1		Attempt complete division by $(x-1)$, or equiv
	A1		Obtain x^2+k
	A1		Obtain completely correct quotient (allow x^2+0x-3)
$x^2=3$	M1		Attempt to solve $x^2=3$
$x=\pm\sqrt{3}$	A1	**6**	Obtain $x=\pm\sqrt{3}$ only
(ii) $\tan x=1, \sqrt{3}, -\sqrt{3}$	B1√		State or imply $\tan x=1$ or $\tan x=$ at least one of their roots from (i)
$\tan x=\sqrt{3}\Rightarrow x={}^{\pi}/_{3}, {}^{4\pi}/_{3}$	M1		Attempt to solve $\tan x=k$ at least once
$\tan x=-\sqrt{3}\Rightarrow x={}^{2\pi}/_{3}, {}^{5\pi}/_{3}$	A1		Obtain at least 2 of ${}^{\pi}/_{3}, {}^{2\pi}/_{3}, {}^{4\pi}/_{3}, {}^{5\pi}/_{3}$ (allow degs/decimals)
$\tan x=1\Rightarrow x={}^{\pi}/_{4}, {}^{5\pi}/_{4}$	A1		Obtain all 4 of ${}^{\pi}/_{3}, {}^{2\pi}/_{3}, {}^{4\pi}/_{3}, {}^{5\pi}/_{3}$ (exact radians only)
	B1		Obtain ${}^{\pi}/_{4}$ (allow degs / decimals)
	B1	**6**	Obtain ${}^{5\pi}/_{4}$ (exact radians only) SR answer only is B1 per root, max of B4 if degs / decimals
	12		