Topic Y7: Binomial and partial fractions (Post-TT) [34] MARKSCHEME

1.

(i) $1+\frac{1}{2}.2x+\frac{\frac{1}{2}.-\frac{1}{2}}{2}\left(4x^2 \text{ or } 2x^2\right)+\frac{\frac{1}{2}.-\frac{1}{2}.-\frac{3}{2}}{6}\left(8x^3 \text{ or } 2x^3\right)$ M1

$=1+x$ B1

$\ldots -\frac{1}{2}x^2+\frac{1}{2}x^3$ (AE fract coeffs) A1 **(3)** For both terms

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(ii) $(1+x)^{-3}=1-3x+6x^2-10x^3$ B1 or $(1+x)^3=1+3x+3x^2+x^3$

Either attempt at their **(i)** multiplied by $(1+x)^{-3}$ M1 or **(i)** long div by $(1+x)^3$

$1-2x\ldots.$ $\sqrt{1+(a-3)x}$ A1 f.t. **(i)** $=1+ax+bx^2+cx^3$

$\ldots +\frac{5}{2}x^2\ldots.$ $\sqrt{(-3a+b+6)x^2}$ A1

$\ldots -2x^3$ $\sqrt{(6a-3b+c-10)x^3}$ A1 **(5)** (AE fract.coeffs)

(iii) $-\frac{1}{2}<x<\frac{1}{2}$, or $|x|<\frac{1}{2}$ B1 **(1)**

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2.

(i) $\left(1+\frac{x}{a}\right)^{-2}=1+(-2)\frac{x}{a}+\frac{-2.-3}{2}\left(\frac{x}{a}\right)^2+\ldots$ M1 Check 3rd term; accept $\frac{x^2}{a}$

$=1-\frac{2x}{a}+\ldots$ or $1+\left(-\frac{2x}{a}\right)$ B1 or $1-2xa^{-1}$ (Ind of M1)

$\ldots +\frac{3x^2}{a^2}+\ldots$ (or $3\left(\frac{x}{a}\right)^2$ or $3x^2a^{-2}$) A1 Accept $\frac{6}{2}$ for 3

$(a+x)^{-2}=\frac{1}{a^2}\left\{\text{their expansion of } \left(1+\frac{x}{a}\right)^{-2}\right\}$ mult out √A1 **4** $\frac{1}{a^2}-\frac{2x}{a^3}+\frac{3x^2}{a^4}$; accept eg a^{-2}

(ii) Mult out $(1-x)$(their exp) to produce all terms/cfs(x^2) M1 Ignore other terms

Produce $\frac{3}{a^2}+\frac{2}{a}(=0)$ or $\frac{3}{a^4}+\frac{2}{a^3}(=0)$ or AEF A1 Accept x^2 if in both terms

$a=-\frac{3}{2}$ www seen anywhere in (i) or (ii) A1 **3** Disregard any ref to $a=0$

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3.

Question	Scheme	Marks	AOs
2 (a)	$(4+5x)^{\frac{1}{2}} = (4)^{\frac{1}{2}}\left(1+\frac{5x}{4}\right)^{\frac{1}{2}} = 2\left(1+\frac{5x}{4}\right)^{\frac{1}{2}}$	B1	1.1b
	$= \{2\}\left[1+\left(\frac{1}{2}\right)\left(\frac{5x}{4}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{5x}{4}\right)^2+\ldots\right]$	M1	1.1b
		A1ft	1.1b
	$= 2+\frac{5}{4}x-\frac{25}{64}x^2+\ldots$	A1	2.1
		(4)	
(b)(i)	$\left\{x=\frac{1}{10}\Rightarrow\right\}(4+5(0.1))^{\frac{1}{2}}$	M1	1.1b
	$=\sqrt{4.5} = \frac{3}{2}\sqrt{2}$ or $\frac{3}{\sqrt{2}}$		
	$\frac{3}{2}\sqrt{2}$ or $1.5\sqrt{2}$ or $\frac{3}{\sqrt{2}} = 2+\frac{5}{4}\left(\frac{1}{10}\right)-\frac{25}{64}\left(\frac{1}{10}\right)^2+\ldots$ $\{=2.121\ldots\}$ $\Rightarrow \frac{3}{2}\sqrt{2}=\frac{543}{256}$ or $\frac{3}{\sqrt{2}}=\frac{543}{256} \Rightarrow \sqrt{2}=\ldots$	M1	3.1a
	So, $\sqrt{2}=\frac{181}{128}$ or $\sqrt{2}=\frac{256}{181}$	A1	1.1b
(b)(ii)	$x=\frac{1}{10}$ satisfies $\lvert x\rvert<\frac{4}{5}$ (o.e.), so the approximation is valid.	B1	2.3
		(4)	
			(8 marks)

4.

(i)	$A=3$	B1		For correct value stated
	$C=1$	B1		For correct value stated
	$11+8x \equiv A(1+x)^2+B(2-x)(1+x)+C(2-x)$	M1		AEF; any suitable identity
	e.g. $A-B=0, 2A+B-C=8, A+2B+2C=11$	A1		For any correct (f.t.) equation involving B
	$B=3$	A1	5	
(ii)	$\left(1-\frac{x}{2}\right)^{-1}=1+\frac{x}{2}+\frac{x^2}{4}+\ldots$	B1		s.o.i.
	$(1+x)^{-1}=1-x+x^2-\ldots$	B1		s.o.i.
	$(1+x)^{-2}=1-2x \quad +3x^2-\ldots$	B1,B1		s.o.i.
	Expansion $=\frac{11}{2}-\frac{17}{4}x+\frac{51}{8}x^2+\ldots$	B1	5	CAO. No f.t. for wrong A and/or B and/or C