Topic Y8 Functions and series (Post-TT A) [39] MARKSCHEME

1.

(i)	State $y \leq 2$	**B1**	**1**	[or equiv; allow $<$; allow any letter or none]
(ii)	Show correct process for composition of functions	**M1**		[numerical or algebraic]
	Obtain 0 and hence 2	**A1**	**2**	[and no other value]
(iii)	State a range of values with 2 as one end-point	**M1**		[continuous set, not just integers]
	State $0 < k \leq 2$	**A1**	**2**	[with correct $<$ and $\leq$ now]

2.

(i)	$a+19d=10, \quad a+49d=70$	M1		Attempt to find d from simultaneous equations involving $a+(n-1)d$ or equiv method
	Hence $30d=60 \Rightarrow d=2$	A1		Obtain $d=2$
	$a+(19 \text{ x } 2)=10$ or $a+(49 \text{ x } 2)=70$	M1		Attempt to find a from $a+(n-1)d$ or equiv
	Hence $a=-28$	A1	**4**	Obtain $a=-28$
(ii)	$S=\frac{29}{2}\left(2\times -28+(29-1)\times 2\right)=0$	M1		For relevant use of $\frac{1}{2}n\left(2a+(n-1)d\right)$
		A1	**2**	For showing the given result correctly **AG**
			6	

3.

(i)		Refer to translation and stretch	M1	in either order; ignore details here; allow any equiv wording (such as move or shift for translation) to describe geometrical transformation but not statements such as add 3 to x
		Either State translation in negative x-direction by 3	A1	or state translation by $\begin{pmatrix}-3\\0\end{pmatrix}$; accept horizontal to indicate direction; term 'translate' or 'translation' needed for award of A1
		State stretch by factor 2 in y-direction	A1	or parallel to y-axis or vertically; term 'stretch' needed for award of A1; these two transformations can be given in either order SC: if M0 but details of one transformation correct, award B1 for 1/3 (in Either, Or 1, Or 2 cases)
			[3]	
		Or 1 State stretch by factor $\frac{1}{2}$ in x-direction	A1	or parallel to x-axis; term 'stretch' needed for award of A1
		State translation in negative x-direction by 3	A1 [3]	or state translation by $\begin{pmatrix}-3\\0\end{pmatrix}$; term 'translate' or 'translation' needed for award of A1; these two transformations must be in this order – if details correct for M1A1A1 but order wrong, award M1A1A0
		Or 2 State translation in negative x-direction by 6	A1	or state translation by $\begin{pmatrix}-6\\0\end{pmatrix}$; term 'translate' or 'translation' needed for award of A1
		State stretch by factor $\frac{1}{2}$ in x-direction	A1 [3]	or parallel to x-axis; term 'stretch' needed for award of A1; these two transformations must be in this order – if details correct for M1A1A1 but order wrong, award M1A1A0
(ii)		Either Solve linear eqn/ineq to obtain critical value -6	B1	
		Attempt solution of linear eqn/ineq where signs of x and $2x$ are different	M1	
		Obtain critical value -2	A1	
		Attempt solution of inequality	M1	using table, sketch, …; implied by correct answer or answer of form $a<x<b$ or of form $x<a, x>b$ (where $a<b$); allow $\leq$ here
		Obtain $-6<x<-2$	A1	as final answer; must be $<$ not $\leq$; allow "$x>-6$ and $x<-2$"
			[5]	
		Or Square both sides to obtain $x^2>4(x^2+6x+9)$	B1	or equiv
		Attempt solution of 3-term quadratic eqn/ineq	M1	with same guidelines as in Q2(ii) for factorising and formula
		Obtain critical values -6 and -2	A1	
		Attempt solution of inequality	M1	using table, sketch, …; implied by correct answer or answer of form $a<x<b$ or of form $x<a, x>b$ (where $a<b$); allow $\leq$ here
		Obtain $-6<x<-2$	A1 [5]	as final answer; must be $<$ not $\leq$; allow '$x>-6$ and $x<-2$'

4.

(i)	$100\,000 \times 0.9^3 = 72900$	M1 A1 2	For relevant use of ar^3 or equiv For the correct answer 72900
(ii)	$100\,000 \times 0.9^x = 5000$ Hence $x\log 0.9 = \log 0.05$ So $x = 28.4$, 28 or 29; or $n = 29.4$, 29 or 30 i.e. 30th year / 30 years / year is 2030	B1 M1 A1 A1√ 4	For a correct equation or inequality For complete solution method by logs or trial For correct solution for **their** index – allow integer values either side For correctly linking **their** index to date or number of years
(iii)	Total is $\frac{100000(1-0.9^{30})}{1-0.9} = 957609$	M1 A1√ A1 3 **9**	For relevant use of $\frac{a(1-r^n)}{1-r}$ For correct (unsimplified) statement for their integer n (if no n stated then use their year – 2000) For answer 958000 or better, including decimal

5.

(i)		$(x+4)-2x=(2x-7)-(x+4)$ **OR** $2x+d=x+4 \quad 2x+2d=2x-7$ $2x=15$ $x=7.5$ **[3]**	M1 A1 A1	Attempt to eliminate d to obtain equation in x only Obtain correct equation in just x Obtain $x=7.5$	Equate two expressions for d, both in terms of x Could use $a+(n-1)d$ twice, and then eliminate d Could use $u_1+u_2+u_3=S_3$ or $u_2=\frac{1}{2}(u_1+u_3)$ Allow unsimplified equation A0 if brackets missing unless implied by subsequent working or final answer Any equivalent form Allow from no working or T&I **Alt method:** B1 - state, or imply, $2x+2d=2x-7$, to obtain $d=-3.5$ M1 - attempt to find x from second equation in x and d A1- obtain $x=7.5$
(ii)	(a)	terms are 16, 12, 9 $^{12}/_{16}=0.75$, $^{9}/_{12}=0.75$ common ratio of 0.75 so GP $S_\infty = {}^{16}/_{1-0.75}$ $=64$ **[4]**	B1 B1 M1 A1	List 3 terms Convincing explanation of why it is a GP Attempt use of $^{a}/_{1-r}$ Obtain 64	Ignore any additional terms given Must show two values of 0.75, or unsimplified fractions Must state, or imply, that ratio has been checked twice, using both pairs of consecutive terms No need to show actual division of terms to justify 0.75, so allow eg arrows from one term to the next with 'x0.75' **SR B2** if 16, 12, 9 never stated explicitly in a list but are soi in a convincing method for $r=0.75$ twice Must be correct formula Could be implied by method Allow if used with their incorrect a and/or r Allow if using $a=8$, even if 16 given correctly in list A0 if given as 'approximately 64'
(ii)	(b)	$^{(2x-7)}/_{(x+4)} = {}^{(x+4)}/_{2x}$ $4x^2-14x=x^2+8x+16$ **OR** $2xr=x+4 \quad 2xr^2=2x-7$ $3x^2-22x-16=0$ $(3x+2)(x-8)=0$ $x=-{}^{2}/_{3}$, $x=8$ **[4]**	M1* A1 M1d* A1	Attempt to eliminate r to obtain equation in x only Obtain $3x^2-22x-16=0$ Attempt to solve quadratic Obtain $x=-{}^{2}/_{3}$	Equate two expressions for r, both in terms of x Could use ar^{n-1} twice, and then eliminate r from simultaneous eqns Allow $6x^2-44x-32=0$ Allow $3x^3-22x^2-16x=0$, or a multiple of this Allow any equivalent form, as long as no brackets and like terms have been combined Condone no = 0, as long as implied by subsequent work Dependent on first M1 for valid method to eliminate r See guidance sheet for acceptable methods Allow recurring decimal, but not rounded or truncated Condone $x=8$ also given Allow from no working or T&I