

## Topic Y8 Functions and series (Post-TT B) [40]

1.

Find the exact solutions of the equation  $|6x - 1| = |x - 1|$ . [4]

2.

The curve  $y = \ln x$  is transformed by:

a reflection in the  $x$ -axis,  
followed by a stretch with scale factor 3 parallel to the  $y$ -axis,  
followed by a translation in the positive  $y$ -direction by  $\ln 4$ .

Find the equation of the resulting curve, giving your answer in the form  $y = \ln(f(x))$ . [4]

3.

In a geometric progression, the sum to infinity is four times the first term.

(i) Show that the common ratio is  $\frac{3}{4}$ . [3]

(ii) Given that the third term is 9, find the first term. [3]

(iii) Find the sum of the first twenty terms. [2]

4.

A sequence  $u_1, u_2, u_3, \dots$  is defined by  $u_n = 85 - 5n$  for  $n \geq 1$ .

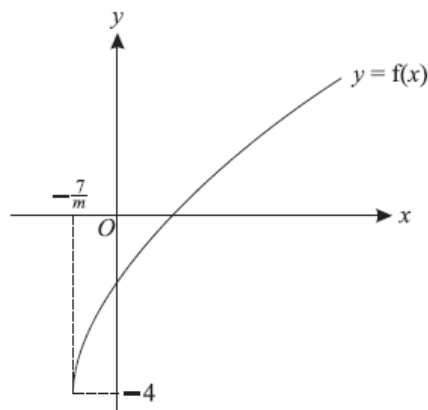
(i) Write down the values of  $u_1, u_2$  and  $u_3$ . [2]

(ii) Find  $\sum_{n=1}^{20} u_n$ . [3]

(iii) Given that  $u_1, u_3$  and  $u_p$  are, respectively, the first, second and third terms of a geometric progression, find the value of  $p$ . [4]

(iv) Find the sum to infinity of the geometric progression in part (iii). [2]

5.



The function  $f$  is defined by  $f(x) = \sqrt{(mx + 7) - 4}$ , where  $x \geq -\frac{7}{m}$  and  $m$  is a positive constant. The diagram shows the curve  $y = f(x)$ .

(i) A sequence of transformations maps the curve  $y = \sqrt{x}$  to the curve  $y = f(x)$ . Give details of these transformations. [4]

(ii) Explain how you can tell that  $f$  is a one-one function and find an expression for  $f^{-1}(x)$ . [4]

(iii) It is given that the curves  $y = f(x)$  and  $y = f^{-1}(x)$  do not meet. Explain how it can be deduced that neither curve meets the line  $y = x$ , and hence determine the set of possible values of  $m$ . [5]