Topic Y8 Functions and series (Post-TT B) [40] MARKSCHEME

1.

	Either Obtain $x = 0$	B1	[ignoring errors in working]
	Form linear equation with signs of $6x$ and x different	M1	[ignoring other sign errors]
	State $6x - 1 = -x + 1$	A1	[or correct equiv with or without brackets]
	Obtain $\frac{2}{7}$ and no other non-zero value	A1	**4** [or exact equiv]
Or	Obtain $36x^2 - 12x + 1 = x^2 - 2x + 1$	B1	[or equiv]
	Attempt to solve quadratic equation	M1	[as far as factorisation or subn into formula]
	Obtain $\frac{2}{7}$ and no other non-zero value	A1	[or exact equiv]
	Obtain 0	B1	**(4)** [ignoring errors in working]

2.

Apply one of the transformations correctly to their equation	B1		
Obtain correct $-3\ln x + \ln 4$	B1		or equiv
Show at least one logarithm property	M1		correctly applied to their equation of resulting curve (even if errors have been made earlier)
Obtain $y = \ln(4x^{-3})$	A1	**4**	or equiv of required form; $\ln 4x^{-3}$ earns A1; correct answer only earns 4/4; condone absence of $y =$
		4	

3.

5 (i)	$4a = \frac{a}{1-r}$	M1	Equate $\frac{a}{1-r}$ to $4a$, or substitute $r = \frac{1}{4}$ into S_∞	S_∞ must be quoted correctly. Allow $4ar^2$ for $4a$. Initially using a numerical value for a is M0. Once equation in a is seen ie $4a = \frac{a}{1-r}$ assume that a has been cancelled if this subsequently becomes $4 = \frac{1}{1-r}$. If initial equation in a is never seen then assume that $a = 1$ is being used and mark accordingly.
	$1 - r = \frac{1}{4}$	M1	Attempt to find value for r or evaluate S_∞	Need to get as far as attempting r. Need to see at least one extra line of working between initial statement and given answer. Substituting numerical value for a is M0 (so M1 M0 possible depending at what stage the substitution happens).
	$r = \frac{3}{4}$	A1	3 Obtain $r = \frac{3}{4}$ (or show $S_\infty = 4a$)	Allow $r = 0.75$.
(ii)	$\left(\frac{3}{4}\right)^2 a = 9$	M1*	Attempt use of ar^2	Must use $r = ¾$ not their incorrect value from (i). Must be clearly intended as ar^2, so $(\frac{3a}{4})^2 = 9$ is M0, unless correct expression previously seen. Can use equivalent method with ratio of $^4/_3$, ie $9 \times (\frac{4}{3})^2$.
	$a = 16$	M1d*	Equate to 9 and attempt to find a	Must get as far as attempting value for a.
		A1	3 Obtain $a = 16$	Answer only gets full credit.
(iii)	$S_{20} = \frac{16\left(1-\left(\frac{3}{4}\right)^{20}\right)}{1-\frac{3}{4}}$ $= 63.8$	M1	Attempt use of correct sum formula for a GP	Must be correct formula, with a = their (a), $r = ¾$ and $n = 20$.
		A1	2 Obtain 63.8, or better	More accurate answer is 63.79704… NB using $n = 1$ rather than n in the formula gives 63.729 (M0), and using $n + 1$ gives 63.848 (M0). Must be decimal, rather than exact answer with power of ¾.
			8	

4.

(i)		$u_1 = 80$ $u_2 = 75$, $u_3 = 70$	**B1** **B1** **[2]**	State 80 State 75 and 70	Just a list of numbers is fine, no need for labels Ignore extra terms beyond u_3
(ii)		$S_{20} = \frac{20}{2}(2 \times 80 + 19 \times -5)$ $= 650$	**M1** **M1** **A1** **[3]**	Show intention to sum 1st 20 terms of an arithmetic sequence Attempt use of correct sum formula for an AP, with $n = 20$, $a = 80$, $d = \pm 5$ Obtain 650	Any recognisable attempt at the sum of an AP, including manual addition of terms – no need to list all of the terms, but intention (inc no of terms) must be clear Must use correct formula – only exception is $10(2a + 9d)$ If using $\frac{1}{2}n(a + l)$, must be a valid attempt at l, either from $a + 19d$ or from u_{20} Answer only gets full marks, as does manual addition
(iii)		$r = \frac{60}{80} = 0.75$ $u_p = 80 \times 0.75^2 = 45$ $85 - 5p = 45$ $p = 8$	**M1*** **A1** **M1d*** **A1** **[4]**	Attempt to find u_p Obtain 45 Attempt to solve $85 - 5p = k$ Obtain $p = 8$	Allow any valid method, inc informal Allow if first and/or second terms of their GP are incorrect Allow ratio of $\frac{4}{3}$ if used correctly to find 3rd term $(60 \div \frac{4}{3})$ Seen or implied **SR: M1* A0** if 45 results from using $u_n = ar^n$. The following M1A1 are still available. k must be from attempt at third term of GP LHS could be $80 + (p - 1)(-5)$, from p^{th} term of the AP, but M0 if incorrect eg $80 + (p - 1)(5)$ Allow full credit for answer only Any variable, including n
(iv)		$S_\infty = \frac{80}{1 - 0.75}$ $= 320$	**M1** **A1** **[2]**	Use correct formula for sum to infinity Obtain 320	Must be from attempt at r for their GP A0 for 'tends to 320', 'approximately 320' etc

5.

(i)	Indicate stretch and (at least one) translation	**M1**	[… in general terms]
	State translation by 7 units in negative x direction	**A1**	[or equiv; using correct terminology]
	State stretch in x direction with factor $1/m$	**A1**	[must follow the translation by 7; or equiv; using correct terminology]
	Indicate translation by 4 units in negative y direction	**B1**	**4** [or equiv; at any stage; the two translations may be combined]
(ii)	Refer to each y value being image of unique x value	**B1**	[or equiv]
	Attempt correct process for finding inverse	**M1**	
	Obtain expression involving $(x+4)^2$ or $(y+4)^2$	**M1**	
	Obtain $\frac{(x+4)^2 - 7}{m}$	**A1**	**4** [or equiv]
(iii)	Refer to fact that curves are reflections of each other in line $y = x$	**B1**	[or equiv]
	Attempt arrangement of either $f(x) = x$ or $f^{-1}(x) = x$	**M1**	
	Apply discriminant to resulting quadratic equati on	**M1**	
	Obtain $(m - 2)(m - 14) < 0$	**A1**	[or equiv]
	Obtain $2 < m < 14$	**A1**	**5**